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FINITE ELEMENT ANALYSIS OF STRESSES

DEFORMATIONS AND PROGRESSIVE FAILURE OF NON-HOMOGENEOUS FISSURED ROCK. VOLUME II. COMPUTER PROGRAMS USER'S MANUAL

OHIO STATE UNIVERSITY

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STRESSES, DEFORMATIONS AND PROGRESSIVE FAILURE OF NON-HOMOGENEOUS FISSURED ROCK

Final Report
Volume 2 – Computer Program User's Manual
March 1973

U.S. BUREAU OF MINES
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Principal Investigators

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The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Advanced Research Projects Agency or the U.S. Government.

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13. ABSTRACT

The objective of this research program was development of finite element procedures for prediction of stresses and deformations in the vicinity of underground excavation.

Two models of rock behavior were selected. In one the rock is treated as isotropic elastic-plastic following a generalized Mohr-Coulomb law and in the other the rock is isotropic elastic-brittle subject to Griffith or modified Griffith failure theory.

For each model, mathematical relationships governing stress-strain behavior and progressive failure were developed. Finite element computer programs incorporating each of the two models were coded. Preliminary to this development, a revised version of Zienkiewicz's no-tension analysis was programmed.

The procedures developed allow for initial stresses in rock, arbitrary shape and size of the opening, any given sequence of construction/excavation, material nonhomogeneity and progressive failure.

This report is in three parts: Volume 1-Main Document; Volume 2-Computer Program User's Manual; Volume 3-Computer Programs.

Volume 2 -Computer Program User's Manual, contains descriptions, instructions for usage and fortran listings of computer programs used to obtain the numerical results presented and discussed in Volume 1 -Main Document.

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FOREWORD

The final report on work done under contract HO210017 between the Ohio State University and the United States Bureau of Mines is in three parts as follows:

Volume 1:	Main Document
Volume 2:	Computer Program User's Manual
Volume 3:	Computer Programs

Volume 2 of the report contains documentation relating to three computer programs including fortran listings and description of program structure.

It is obvious that these computer programs be used only under the conditions and assumptions for which they were developed. These are described in Volume 1 of this report. Although the programs have been tested by applications to several problems, no warranty is made regarding the accuracy and reliability of the programs and no responsibility is assumed by the authors or by the sponsors of this research project.

The technical report summary is included in all three volumes of the report.

R. S. Sandhu
Principal Investigator

TECHNICAL REPORT SUMMARY

Program Objectives

The objective of this research program was development of finite element procedures to predict stresses, deformations and progressive failure of rock associated with underground excavations. For applicability to arbitrary sequence of excavation operations, it was necessary that the procedures developed allow for arbitrary initial stresses in rock, arbitrary size and shape of the opening and progressive failure. Plane strain conditions and two different types of material behavior were considered. Rock was treated as an isotropic elastic-plastic generalized Mohr-Coulomb material in one model and as an elastic-brittle material following Griffith theory of fracture in the other.

Background

In previous applications of the finite element method to rock mechanics, elastic-plastic behavior of rock has been modeled as nonlinear elastic for computational convenience. Further, it was assumed that the results of a one-dimensional test could be generalized to three-dimensional analysis through the use of an equivalent stress-equivalent strain curve. In some applications, two stress or strain parameters were used. These procedures are unsatisfactory. Assumption of isotropic elasticity assumes that the principal directions of stress and strain coincide. In plasticity this is not true. Also, rock behavior is characterized by a significant part of deformation being irreversible. For this reason, the mechanical behavior in unloading is different from that in loading. For rock with preexisting joints or developing tensile cracks, a 'no tension' procedure is often adopted. In this method, a linear elastic solution is obtained and all tensile stress redistributed simultaneously. Actually, as cracking progresses, the rock on either side of the crack is relieved of stress and a stress concentration develops near the crack tip. Conventional procedures ignore these effects and the progressive nature of crack development, leading to erroneous conclusions regarding stresses around underground openings.

Accomplishments Under the Present Program

The research conducted under this contract has resulted in development of computer programs based on more realistic simulation of material behavior. The incremental theory of plasticity has been used to characterize the stress-strain behavior of elastic-plastic rock. Role of kinematic constraint of plane strain in development of residual stresses in rock has been examined on the basis of Hill's theory. New techniques have been developed for study of initiation and propagation of fracture in rock following Griffith's theory or the modified Griffith theory. Allowing for sequential fracture of various elements in a system, the effect of progressive stress redistribution in the remaining system is correctly incorporated. Arbitrary initial stress states, arbitrary sequence of excavation (or construction), arbitrary size and shape of opening, and nonhomogeneous material properties were allowed for. The actual construction operations can be simulated. The procedures developed were applied to several typical problems in rock mechanics as well as to some theoretical and laboratory studies for the purpose of verification and illustration. These were used to carry out parametric studies to examine the influence of rock properties upon the stresses in steel supports in a tunnel.

DOD Implications

The procedures developed provide useful means for study of stability of underground excavations based on stresses and deformations associated with the mining operations, structural support evaluation, safety analyses of openings, study of blasting effectiveness under certain conditions, evaluation of mining sequences, study of vulnerability and serviceability of underground structures etc.

Organization of the Report

This report is in three parts as follows:

Volume 1 - Main Document
Volume 2 - Computer Program User's Manual
Volume 3 - Computer Programs

Volume 1 contains the main body of the report including the theoretical development, program verification and case studies. Chapter I reviews previous efforts in the general research area and describes the objectives and methods of the present research in the historical context. Chapter II describes the mechanical behavior of rock and the idealizations used in the research under report. The basis and methods of the finite element theory are briefly discussed in Chapter III leading to the formulation of matix equations. Chapter IV gives details of the analysis technique for isotropic elastic-plastic generalized Mohr-Coulomb rock materials and Chapter V gives the numerical analysis procedure for jointed rock and rock subjected to progressive fracture following Griffith or modified Griffith theory. Examples of application are included in Chapters IV and V. Chapter VI presents application of the elastic-plastic analysis computer program to a parametric study to evaluate the influence of rock properties on stresses in steel supports for specified initial stresses and design of the opening.

In the original proposal, model testing to verify some aspects of rock behavior under plane strain conditions was foreseen. The effort under the present contract covered procurement of suitable plane strain test equipment and design of suitable test material. Appendix B includes a report on this effort.

Volume 2 of the report contains description of the three computer programs developed under the contract along with fortran listings and instructions for input preparation. The input definition and the listings are for the IBM 370/165 version.

The programs are the primary content of volume 3. These are available on magnetic tape from DDC-TC, U.S. Department of Commerce, Springfield, Virginia 22151, telephone (703) 321-8517.

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The research was supported by the U.S. Government through the Advanced Research Projects Agency, ARPA, and its agent the U.S. Bureau of Mines, Department of the Interior. James J. Olson, Twin Cities Mining Research Center, was the ARPA program coordinator and Dr. William J. Karwoski, Spokane Mining Research Center, was the Project Officer. In early stages of work. Dr. Syd Peng, Twin Cities Mining Research Center acted as the Project Officer. Constant cooperation and several constructive suggestions from these individuals are gratefully appreciated.

A number of graduate students worked on the project. The contributions of Messrs. Ram Dhan Singh, S. W. Huang and Kamar Jit Singh were specially noteworthy. The Instruction and Research Computer Center of the Ohio State University provided the computational facilities.

R. S. Sandhu
Project Supervisor

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CHAPTER I: NOTENS-- A Computer Program for Analysis of Roek as a No-Tension Material

1.1. Purpose and Capability

This eomputer program is based on a modifieation of Zienkiewiez's approach (Zienkiewiez et.al. 1968). A linear elastic analysis of a given two-dimensional system is earried out to determine the regions of tensile stress. The tensions, wherever they occur, are replaeed by equivalent nodal point loads and the tensile stresses in the corresponding elements are neutralized. For these equivalent loads, the elastie analysis is repeated. This iterative process is eontinued until all tensile stresses are liquidated. The modifieation to the original approach is diseussed in Part I: Technieal Report.

The program allows for an arbitrary initial stress field, gravity loads, eoncentrated loads, distributed loads applied to the boundary, temperature loads and temperature dependent material properties.

1.2. Organization

The program is in Fortran language. Scrateh tapes are 1 and 2. Tapes 5 and 6 are used for input/output respectively. The listing in section 1.6 used double precision for real numbers. The program eapacity ean be altered by ehanging the dimension of arrays AA and IA. These correspond to the total locations required in real arrays and integer arrays respeetively. NTOT, MTOT at lines MAIN 23, MAIN 24 are set equal to the dimension of AA, IA respectively.

The program eonsists of the following units:

a. Program MAIN

In this unit, the eontrol information is read in and the loeation of various dimensioned variables defined in the system arrays AA and IA.

b. Subroutine INPT

This subroutine is called by the MAIN. In this, information regarding material properties; nodal point coordinates, loads, temperatures, boundary conditions; element geometry, material and initial stresses is read in. Missing information is generated. Information regarding boundary pressure is read in. Dimensions of the matrix for solution of stiffness equations are defined and subroutine SOLVE is called to complete the solution proeess.

c. Subroutine SOLVE

Called by INPT after reading in all data, this subroutine sets up the system stiffness matrix in blocks. It ealls subroutines ONED and ELEMEN to obtain stiffness properties for one-dimensional and two-dimensional elements respectively. The stiffness is assembled by the direet stiffness procedure and modified for specified boundary displacements. The matrix is triangularized by ealling SYMBAN (1). Subroutine LOAD is called to recover the system load veetor. Reduetion of the load vector and baek-substitution are aeeomplished by calling SYMBAN (2). The cumulative as well as the inerenental displacements are pointed out. Subroutine STRESS is called for evaluation and print out of stresses. Because the solution is obtained by elimination of tension in an iterative sequenee, the steps from calling LOAD to caleulation of stress are repeated a preset number of times or until convergenee is obtained.

d. Subroutine ONED

Called by SOLVE, this subroutine sets up the stiffness matrix for a one-dimensional element.

e. Subroutine ELEMEN

Called by SOLVE, this subroutine calculates the stiffness matrix for two-dimensional elements. These maybe triangular elements or quadrilaterals made up of four constant strain triangles. If other types of quadrilateral elements are to be used, this subroutine has to be replaced. Linear isotropic elasticity is assumed. Loads due to initial stresses or temperature changes from a reference temperature, and gravity loads are calculated in this subroutine.

f. Subroutine MODIFY

Called by SOLVE, this subroutine modifies the system stiffness matrix to allow for prescribed displacement boundary conditions.

g. Subroutine SYMBAN

This subroutine is called by SOLVE. Called with augment 1, it triangularizes the stiffness matrix and called with augment 2, it backsubstitutes to evaluate the solution for displacements for a given load vector.

h. Subroutine LOAD

This subroutine called by SOLVE, sets up the system load vector including nodal point loads, distributed boundary pressure loads, as well as unbalanced element loads, initial stresses, temperature stresses and gravity loads.

j. Subroutine STRESS

Stresses in x-y coordinate system as well as principal stresses are calculated in the STRESS subroutine called by SOLVE. The stresses are checked for tension, and if necessary, the tensions are replaced by equivalent unbalanced stresses to be transformed into load in the next cycle. A check on convergence of the iterative scheme is applied and the maximum unbalanced stress in any element is printed out.

1.3. Input Data

Input to the program NOTENS consists of the following sequence of cards.

a. First Card. Job Title (18A4).

This card gives the descriptive identification of the job.

b. Second Card. Control Information (4I5, 3F10.2, I5, D15.4)

Information	Columns
Total number of nodal points (NUMNP)	1-5
Total number of elements (NUMEL)	6-10
Number of different materials (NUMMAT)	11-15
Number of pressure boundary cards (NUMPL)	16-20
Acceleration in x-direction (ACELR)	21-30
Acceleration in y-direction (ACELZ)	31-40
Reference (stress-free) temperature (Q)	41-50
Maximum number of approximations or iterations allowed (NP)	51-55
Tolerance for convergence of iterations (TOL)	56-70

c. Material Property Cards

One set of cards must be provided for each material. In each set:

i. First card (2I5, 1F10.3, I5) gives the following information:

Material identification number ($1 \leq$ MTYPE ≤ 10)	1-5
Number of temperature cards (NTC)	6-10

Mass density of the material (RO)	11-20
Material code (MTC): 1 for no-tension material 0 otherwise	21-25

ii. Subsequent cards, one for each temperature, the number of data sets defined in columns 6-10 of the first card for the material, will carry the the following information (4F10.3):

Information	Columns
Temperature	1-10
Elastic Modulus	11-20
Poisson's ratio	21-30
Coefficient of thermal expansion (or cross-sectional area of 1-D element)	31-40

d. Nodal Point Cards (I5, F5.1, 5F10.4)

Nodal point number (N)	1-5
Code of nodal point (CODE)	6-10
X-ordinate (R)	11-20
Y-ordinate (Z)	21-30
UR	31-40
UZ	41-50
Temperature (T)	51-60

If the number in columns 6-10 is

0. UR is the specified X-load and UZ is the specified Y-load

1. UR is the specified X-displacement and UZ is the specified Y-load

2. UR is the specified X-load and UZ is the specified Y-displacement

3. UR is the specified X-displacement and UZ is the specified Y-displacement

All loads are considered to be total forces acting on an element of unit thickness. Nodal point cards must be in numerical sequence. If cards are omitted, the omitted nodal points are generated at equal intervals along a straight line between the defined nodal points. Similarly, the corresponding temperatures are determined by linear interpolation. The codes (CODE) of these generated nodal points, as well as UR and UZ, are set equal to 0.

e. Element cards

i. Material type information of elements (16I5)

Every 5 columns of each data card give the material identification number of each element in sequence. Each card contains 16 material identification numbers (80 columns of 16 elements).

ii. Nodal points and initial stresses (5I5, 5X, 3F10.0). One card for each element.

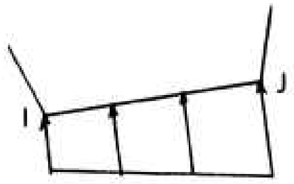
Information	Columns
Number of the element	1-5
Nodal point I	6-10
Nodal point J	11-15
Nodal point K	16-20
Nodal point L	21-25
Initial σ_{xx}	31-40
Initial σ_{yy}	41-50
Initial σ_{xy}	51-60

Nodal points I, J, K, L are corners of each individual element in a counterclockwise order for a right-handed system of co-ordinates. For 1-D elements, nodal points K and L are set same as J and I respectively. In the case of triangular elements, L is set to be same as K. The element cards must be in numerical sequence. Any element cards omitted will be automatically generated in the program by incrementing each of the I, J, K and L nodal points by 1. The corresponding initial stresses of the generated elements are calculated by the program using linear interpolation. Such interpolation is done basing upon element number rather than distance.

f. Pressure Boundary cards (2I5, 2F10.3)

One card for each boundary element, which is subjected to a normal pressure, will carry the following information:

Information	Columns
Nodal point I (IBC)	1-5
Nodal point J (JBC)	6-10
Normal pressure at I (PR)	11-20
Normal pressure at J (PR)	21-30



As shown in the sketch, the boundary element must be on the left as one progresses counter-clockwise from I to J. Surface tensile force is input as a negative pressure.

1.4. Output Data

The following information is developed and printed by the program:

a. Printing of input (and generated)data

b. Incremental and cumulative nodal displacements of each iteration

c. Stresses at center of each element and the unbalanced force of the elements during each iteration

d. Maximum unbalanced force of the elements for the iteration

e. In the case where maximum unbalanced force is less than the set tolerance (TOL), convergence is noted and the number of cycles to reach this is printed.

1.5. Limitations

Limitations of the approach are discussed in detail in Part I-Technical Report. In this approach it is assumed that all the elements having tensile stress crack and are relieved of tension simultaneously. This is unrealistic. Actually the process of cracking will be sequential with cracking of each element influencing the stress distribution and consequently the continuation of the cracking sequence. The program included here eliminates some of the procedural errors of earlier attempts. However, it cannot predict sequential or progressive development of cracks. Also convergence is often very slow and the advantage of using the same system stiffness matrix throughout the iterative computation is lost because of the large number of iterations required.

1.6. Fortran Listing

```
C     ***************************************************************
C     PROGRAM NOTENS
C     NO TENSION ANALYSIS FOR PLANE STRESS AND PLANE STRAIN.
C     LINEAR PRESSURE BOUNDARY
C     PROGRAMMED BY R.S.SANDHU, R.D.SINGH AND H.LIU. THE OHIO STATE
C     UNIVERSITY, COLUMBUS.
C     THE FORMULATION IS DOCUMENTED IN THE FINAL REPORT DATED MARCH 31,
C     1973, ON CONTRACT H0210017 BETWEEN THE OHIO STATE UNIVERSITY AND
C     THE UNITED STATES BUREAU OF MINES SUPPORTED BY THE ADVANCED
C     RESEARCH PROJECTS AGENCY. INSTRUCTIONS FOR USE OF THE PROGRAM
C     ARE CONTAINED IN PART II OF THE REPORT.
C     ***************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON AA(10000),IA(1500)
      COMMON/ONE/ACELR,ACELZ,TEMP,W,TOL,VOL,NUMNP,NUMEL,NUMMAT,NUMPC,
     *MTYPE,LLL,N,MBAND,NUMBLK,KKK,NCHECK,NP,NL,NEQ,HED(18),NPC
      COMMON/TWO/C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),XC,YC,
     *EE(3),LM(4),E(5,4,10),RC(10),NTC(10),MTC(10)
      COMMON/THREE/MTOT,NTOT,N11,M4,STOP
      READ(5,1000) HED,NUMNP,NUMEL,NUMMAT,NUMPC,ACELR,ACELZ,Q,NP,TOL
      WRITE(6,2000)HED,NUMNP,NUMEL,NUMMAT,NUMPC,ACELR,ACELZ,Q,NP,TOL
      NTOT=10000
      MTOT=1500
      NPC=NUMPC
      IF(NUMPC.EQ.0)NPC=1
      N1=1
      N2=N1+NUMNP
      N3=N2+NUMNP
      N4=N3+NUMNP
      N5=N4+NUMNP
      N6=N5+NUMNP
      N7=N6+2*NPC
      N8=N7+6*NUMEL
      N9=N8+NUMNP
      N10=N9+2*NUMNP
      N11=N10+2*NUMNP
      M1=1
      M2=M1+5*NUMEL
      M3=M2+NPC
      M4=M3+NPC
      NEQ=2*NUMNP
      JJ=M4-MTOT
      IF(JJ.LE.0)GO TO 100
      WRITE(6,3000)JJ
      CALL EXIT
  100 CONTINUE
      CALL INPT(AA(N1),AA(N2),AA(N3),AA(N4),AA(N5),AA(N6),AA(N7),
     *AA(N8),AA(N9),AA(N10),IA(M1),IA(M2),IA(M3))
 1000 FORMAT(18A4/4I5,3F10.2,I5,D15.4)
 2000 FORMAT (1H1 18A4/
     1 30H0 NUMBER OF NODAL POINTS------ I3 /
     2 30H0 NUMBER OF ELEMENTS---------- I3 /
     3 30H0 NUMBER OF DIFF. MATERIALS--- I3 /
     4 30H0 NUMBER OF PRESSURE CARDS---- I3 /
     5 30H0 X-ACCELERATION-------------- E12.4/
     6 30H0 Y-ACCELERATION-------------- E12.4/
     7 30H0 REFERENCE TEMPERATURE------- E12.4/
     8 30H0 NO. OF APPROXIMATIONS------- I5/
     9 30H0 TOLERANCE FOR CONVERGENCE--- E12.4)
 3000 FORMAT (70H PROGRAM EXECUTION TERMINATED. REQUIRED CORE EXCEEDS NT
     *OT BY                                                          I10)
      END
      SUBROUTINE INPT(R,Z,UR,UZ,T,PR,SIGI,CODE,B,CU,IX,IBC,JBC)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON AA(10000),IA(1500)
      COMMON/ONE/ACELR,ACELZ,TEMP,Q,TOL,VOL,NUMNP,NUMEL,NUMMAT,NUMPC,
     *MTYPE,LLL,N,MBAND,NUMBLK,KKK,NCHECK,NP,NL,NEQ,HED(18),NPC
      COMMON/TWO/C(3,3),S(10,10),SIG(6),P(6),ST(3,10),RR(5),ZZ(5),XC,YC,
     *EE(3),LM(4),E(5,4,10),RO(10),NTC(10),MTC(10)
      COMMON/THREE/MTOT,NTOT,N11,M4,SIOP
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),T(NUMNP),PP(NPC,2)
     *,SIG1(NUMEL,6),CODE(NUMNP),IX(NUMEL,5),IBC(NPC),JBC(NPC)
      DIMENSION B(NEQ),CU(NEQ)
      DO 50 M=1,NUMMAT
      READ (5,1001) MTYPE,NTC(MTYPE),RO(MTYPE),MTC(MTYPE)
      WRITE(6,2001) MTYPE,NTC(MTYPE),RO(MTYPE),MTC(MTYPE)
      NUMTC=NTC(MTYPE)
      READ (5,1002) ((E(I,J,MTYPE),J=1,4),I=1,NUMTC)
      WRITE (6,2002) ((E(I,J,MTYPE),J=1,4),I=1,NUMTC)
   50 CONTINUE
      WRITE (6,2003)
      L=0
   60 READ  (5,1003) N,CODE(N),R(N),Z(N),UR(N),UZ(N),T(N)
      NL=L+1
      IF(N.EQ.1) GO TO 70
      ZX=N-L
      DR=(R(N)-R(L))/ZX
      DZ=(Z(N)-Z(L))/ZX
      DT=(T(N)-T(L))/ZX
   70 L=L+1
      IF(N-L) 100,90,80
   80 CODE(L)=0.0
      R(L)=R(L-1)+DR
      Z(L)=Z(L-1)+DZ
      T(L)=T(L-1)+DT
      UR(L)=0.0
      UZ(L)=0.0
      GO TO 70
   90 CONTINUE
      IF(NUMNP-N) 100,110,60
  100 WRITE (6,2005) N
      CALL EXIT
  110 WRITE(6,2004)((K,CODE(K),R(K),Z(K),UR(K),UZ(K),T(K)),K=1,NUMNP)
      READ(5,1007) (IX(N,5),N=1,NUMEL)
      WRITE (6,2006)
      N=0
  130 READ (5,1004) M,(IX(M,I),I=1,4) ,(SIGI(M,I),I=1,3)
      IF(M.EQ.1) GO TO 140
      ZX=M-N
      DO 135 I=1,3
  135 SIG(I) =(SIGI(M,I)-SIGI(N,I)) / ZX
  140 N=N+1
```

```
      IF(M.LE.N)GO TO 170
      IX(N,1)=IX(N-1,1)+1
      IX(N,2)=IX(N-1,2)+1
      IX(N,3)=IX(N-1,3)+1
      IX(N,4)=IX(N-1,4)+1
      DO 160 I=1,3
  160 SIGI(N,I)=SIGI(N-1,I)+SIG(I)
  170 IF(M.GT.N) GO TO 140
      IF(N.LT.NUMEL)GO TO130
      WRITE(6,2007)((N,(IX(N,I),I=1,5),(SIGI(N,I),I=1,3)),N=1,NUMEL)
      IF(NUMPC.EQ.0) GO TO 310
  290 WRITE (6,2008)
      DO 300 L=1,NUMPC
      READ(5,1005) IBC(L),JBC(L),PR(L,1),PR(L,2)
  300 WRITE(6,2009) IBC(L),JBC(L),PR(L,1),PR(L,2)
  310 CONTINUE
      J=0
      DO 340 N=1,NUMEL
      DO 340 I=1,4
      DO 325 L=1,4
      KK=IABS(IX(N,I)-IX(N,L))
      IF(KK.GT.J) J=KK
  325 CONTINUE
  340 CONTINUE
      MBAND=2*J+2
      WRITE(6,3000)MBAND
      NL=(NTOT-N11+1)/MBAND
      NLL=NEQ+3
      IF(NL.GT.NLL) NL=NLL
      NL=NL/4
      NL=4*NL
      NBAND=2*MBAND
      IF(NL.GE.NBAND)GO TO 350
      WRITE (6,4010)NL,MBAND
      CALL EXIT
  350 CONTINUE
      N12=N11+NL*MBAND
      JJ=N12-NTOT
      IF(JJ.LE.0)GO TO 360
      WRITE(6,3050)JJ
      CALL EXIT
  360 CONTINUE
      WRITE(6,4000)N12,M4
          CALL SOLVE(R,Z,UR,UZ,T,PR,SIGI,CODE,B,CO,AA(N11),IX,
     *IBC,JBC)
      RETURN
 1001 FORMAT (2I5,1F10.3,I5)
 1002 FORMAT (4F10.3)
 1003 FORMAT (I5,F5.1,5F10.4)
 1004 FORMAT(5I5,5X,3F10.0)
 1005 FORMAT(2I5,2F10.3)
 1007 FORMAT(16I5)
 2001 FORMAT (17H0MATERIAL NUMBER= I3, 30H, NUMBER OF TEMPERATURE CARDS=
     1 I3, 15H, MASS DENSITY= F12.4,16H, MATERIAL CODE= I5)
 2002 FORMAT (15H0  TEMPERATURE 10X 5HE      9X 6HNU      10X 5HALPHA/
     1(F15.2,3E15.5))
 2003 FORMAT (108H1NODAL POINT         TYPE   X ORDINATE   Y ORDINATE   X LO
     1AD OR DISPLACEMENT  Y LOAD OR DISPLACEMENT TEMPERATURE )
 2004 FORMAT (I12,F12.2,2F12.5,2E24.7,F12.3)
 2005 FORMAT (26H0NODAL POINT CARD ERROR N= I5)
 2006 FORMAT(108H1ELEMENT NO.       I      J      K      L     MATERIAL
     1SIGIXX         SIGIYY         SIGIXY                                 )
 2007 FORMAT (I113,4I6,I112,3F12.3)
 2008 FORMAT(29H0PRESSURE BOUNDARY CONDITIONS/40H      I      J     PRESSUR
     1E I     PRESSURE J)
 2009 FORMAT(2I6,2F14.3)
 3000 FORMAT(30H BAND WIDTH------ -----------                          I5)
 3050 FORMAT (70H PROGRAM EXECUTION TERMINATED. REQUIRED CORE EXCEEDS NT
     *OT BY                                                          I10)
 4000 FORMAT (47H FOR THIS PROGRAM THE LOCATION USED IN AA IS =      I5,
     *17H AND IN IA IS =           I5)
 4010 FORMAT (25H0 NL IS LESS THAN 2*MBAND                             /
     *5H0 NL=                                                          I5/
     *8H0 MBAND=                                                       I5)
      END
      SUBROUTINE SOLVE(R,Z,UR,UZ,T,PR,SIGI,CODE,B,CU,A,IX,IBC,JBC)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON AA(10000),IA(1500)
      COMMON/ONE/ACELR,ACELZ,TEMP,Q,TOL,VOL,NUMNP,NUMEL,NUMMAT,NUMPC,
     *MTYPE,LLL,N,MBAND,NUMBLK,KKK,NCHECK,NP,NL,NEQ,HED(18),NPC
      COMMON/TWO/C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),XC,YC,
     *EE(3),LM(4),E(5,4,10),RO(10),NTC(10),MTC(10)
      COMMON/THREE/MTOT,NTOT,N11,M4,STOP
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),T(NUMNP),PR(NPC,2)
     *,SIGI(NUMEL,6),CODE(NUMNP),IX(NUMEL,5),IBC(NPC),JBC(NPC)
      OIMENSION CU(NEQ),A(NL,MBANO),B(NEQ)
      DO 10 N=1,NUMEL
      OO 10 I=4,6
   10 SIGI(N,1)=0.
      OO 20 N=1,NUMNP
      NN=2*N
      CU(NN-1)=0.
   20 CU(NN)=0.
      REWINO 2
      ND2=NL
      NO=NL/2
      NB=NO/2
      STOP=0.0
      NUMBLK=0
      DO 50 N=1,ND2
      OO 50 M=1,MBAND
   50 A(N,M)=0.0
   60 NUMBLK=NUMBLK+1
      NH=NB*(NUMBLK+1)
      NM=NH-NB
      NZ=NM-NB+1
      KSHIFT=2*NZ-2
      DO 210 N=1,NUMEL
      NN=IX(N,1)
      DO 65 I=2,4
      IF(IX(N,I).LT.NN) NN=IX(N,I)
   65 CONTINUE
      IF((NN.LT.NZ).OR.(NN.GT.NM)) GO TO 210
      IF(IX(N,3).NE.IX(N,2)) GO TO 95
      CALL ONED(R,Z,UR,UZ,T,PR,SIGI,CODE,CU,A,B,IX,IBC,JBC)
      MM=2
      GO TO 130
   95 CALL ELEMEN (R,Z,UR,UZ,T,PR,SIGI,CODE,CU,A,B,IX,IBC,JBC)
      IF(VOL.GT.0)GO TO 110
  100 WRITE(6,2000) N
      STOP=1.0
  110 MM=4
      IF(IX(N,3).EQ.IX(N,4))MM=3
  130 CONTINUE
      DO 140 I=1,MM
  140 LM(I)=2*IX(N,I)-2
      DO 200 I=1,MM
      DO 200 K=1,2
      II=LM(I)+K-KSHIFT
      KK=2*I-2+K
      DO 200 J=1,MM
      DO 200 L=1,2
      JJ=LM(J)+L-II+1-KSHIFT
      LL=2*J-2+L
      IF(JJ.LE.0) GO TO 200
      IF(ND.GE.JJ) GO TO 195
  180 WRITE (6,2001) N
      STOP=1.0
      GO TO 210
  195 A(II,JJ)=A(II,JJ)+S(KK,LL)
  200 CONTINUE
  210 CONTINUE
      DO 400 M=NZ,NH
      IF(M.GT.NUMNP) GO TO 410
      N=2*M-1-KSHIFT
      IF(CODE(M)) 390,400,316
  316 IF(CODE(M).EQ.1.)GO TO 370
      IF(CODE(M).EQ.2.)GO TO 390
      GO TO 380
  370 CALL MODIFY(A,ND2,MBAND,N)
      GO TO 400
  380 CALL MODIFY(A,ND2,MBAND,N)
  390 CONTINUE
      N=N+1
      CALL MODIFY(A,ND2,MBAND,N)
  400 CONTINUE
  410 CONTINUE
      WRITE(2) ((A(N,M),M=1,MBAND),N=1,ND)
      DO 420 N=1,ND
      K=N+ND
      DO 420 M=1,MBAND
      A(N,M)=A(K,M)
  420 A(K,M)=0.0
      IF(NM.LT.NUMNP) GO TO 60
      IF(STOP.NE.0.)CALL EXIT
      NCHECK=1
      KKK=1
      CALL SYMBAN(ND2,A,B,NEQ,MBAND,NUMBLK,KKK)
      KKK=2
      DO 550 LLL=1,NP
      CALL LOAD(R,Z,UR,UZ,T,PR,SIGI,CODE,CU,A,B,IX,IBC,JBC)
      CALL SYMBAN(ND2,A,B,NEQ,MBAND,NUMBLK,KKK)
      DO 510 N=1,NUMNP
      NN=2*N
      CU(NN-1)=CU(NN-1)+B(NN-1)
  510 CU(NN)=CU(NN)+B(NN)
      WRITE (6,2013) LLL
      WRITE(6,2010) (N,B(2*N-1),B(2*N),CU(2*N-I),CU(2*N), N = I,NUMNP)
      CALL STRESS(R,Z,UR,UZ,T,PR,SIGI,CODE,CU,A,B,IX,IBC,JBC)
      IF(NCHECK.EQ.0) GO TO 600
  550 CONTINUE
      GO TO 990
  600 WRITE (6,2011)LLL
  990 RETURN
 2000 FORMAT (26HONEGATIVE AREA ELEMENT NO. I4)
 2001 FORMAT (29HOBAND WIDTH EXCEEDS ALLOWABLE I4)
 2010 FORMAT(12HON.P.NUMBER 17X 3HDUX 17X 3HDUY I8X 2HUX 18X 2HUY/
     1(1I12,4E20.7))
 2011 FORMAT(35HO NUMBER OF CYCLES TO CONVERGENCE =    I5)
 2013 FORMAT(30H1 RESULTS OF ITERATION  NO.=      I5///)
      END
      SUBROUTINE ONED(R,Z,UR,UZ,T,PR,SIGI,CODE,CU,A,B,IX,IBC,JBC)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON AA(10000),IA(1500)
      COMMON/ONE/ACELR,ACELZ,TEMP,Q,TOL,VOL,NUMNP,NUMEL,NUMMAT,NUMPC,
     *MTYPE,LLL,N,MBAND,NUMBLK,KKK,NCHECK,NP,NL,NEQ,HED(18),NPC
      COMMON/TWO/C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),XC,YC,
     *EE(3),LM(4),E(5,4,10),RO(10),NTC(10),MTC(10)
      COMMON/THREE/MTOT,NTOT,NI1,M4,STOP
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),T(NUMNP),PR(NPC,2)
     *,SIGI(NUMEL,6),CODE(NUMNP),IX(NUMEL,5),IBC(NPC),JBC(NPC)
C
C
      DO 100 I=1,8
      P(I)=0.0
      DO 100 J=1,8
  100 S(I,J)=0.0
      MTYPE=IX(N,5)
      I=IX(N,1)
      J=IX(N,2)
      DX=R(J)-R(I)
      DY=Z(J)-Z(I)
      XL=DSQRT(DX**2+DY**2)
      COSA=DX/XL
      SINA=DY/XL
      COMM=E(1,2,MTYPE)*E(1,4,MTYPE)/XL
C
      S(1,1)=COSA*COSA*COMM
      S(1,2)=COSA*SINA*COMM
      S(1,3)=-S(1,1)
      S(1,4)=-S(1,2)
      S(2,1)=S(1,2)
      S(2,2)=SINA*SINA*COMM
      S(2,3)=-S(1,2)
      S(2,4)=-S(2,2)
      S(3,1)=S(1,3)
      S(3,2)=S(2,3)
      S(3,3)=S(1,1)
      S(3,4)=S(1,2)
      S(4,1)=S(1,4)
      S(4,2)=S(2,4)
      S(4,3)=S(3,4)
      S(4,4)=S(2,2)
C
C
      RETURN
C
      END
      SUBROUTINE ELEMEN(R,Z,UR,UZ,T,PR,SIGI,CODE,CU,A,B,IX,IBC,JBC)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON AA(10000),IA(1500)
      COMMON/ONE/ACELR,ACELZ,TEMP,Q,TOL,VOL,NUMNP,NUMEL,NUMMAT,NUMPC,
     *MTYPE,LLL,N,MBAND,NUMBLK,KKK,NCHECK,NP,NL,NEQ,HED(18),NPC
      COMMON/TWO/C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),XC,YC,
     *EE(3),LM(4),E(5,4,10),RO(10),NTC(10),MTC(10)
      COMMON/THREE/MTOT,NTOT,N11,M4,STOP
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),T(NUMNP),PR(NPC,2)
     *,SIGI(NUMEL,6),CODE(NUMNP),IX(NUMEL,5),IBC(NPC),JBC(NPC)
      DIMENSION CU(NEQ),A(NL,MBAND),B(NEQ)
      DIMENSION U(3),V(3)
C
      I=IX(N,1)
      J=IX(N,2)
      K=IX(N,3)
      L=IX(N,4)
      MTYPE=IX(N,5)
      VOL=0.
      TEMP=(T(I)+T(J)+T(K)+T(L))/4.0
      RATIO=0.0
      NUMTC=NTC(MTYPE)
      IF (NUMTC.EQ.1) GO TO 100
      DO 50 M=2,NUMTC
      IF (E(M,1,MTYPE)-TEMP) 50,60,60
   50 CONTINUE
   60 DEN=E(M,1,MTYPE)-E(M-1,1,MTYPE)
      IF(DEN.EQ.0.) GO TO 80
      RATIO=(TEMP-E(M-1,1,MTYPE))/DEN
   80 DO 90 KK=1,3
   90 EE(KK)=E(M-1,KK+1,MTYPE)+RATIO*(E(M,KK+1,MTYPE)-E(M-1,KK+1,MTYPE))
      GO TO 110
  100 DO 105 KK=1,3
  105 EE(KK)=E(1,KK+1,MTYPE)
  110 COMM=EE(1)/(1.-EE(2)**2)
      C(1,1)=COMM
      C(1,2)=COMM*EE(2)
      C(1,3)=0.
      C(2,1)=C(1,2)
      C(2,2)=C(1,1)
      C(2,3)=0.
      C(3,1)=0.
      C(3,2)=0.
      C(3,3)=.5*COMM*(1.-EE(2))
      DO 130 J=1,10
      DO 120 I=1,3
  120 ST(I,J)=0.
      DO 130 I=1,10
  130 S(I,J)=0.
      DO 140 I=1,4
      NPP=IX(N,1)
      RR(1)=R(NPP)
  140 ZZ(1)=Z(NPP)
      IF(IX(N,3).EQ.IX(N,4))GO TO 150
      XC=(RR(1)+RR(2)+RR(3)+RR(4))/4.
      YC=(ZZ(1)+ZZ(2)+ZZ(3)+ZZ(4))/4.
      RR(5)=XC
      ZZ(5)=YC
      K=5
      J=1
      I=4
      LM(3)=9
      NT=4
      GO TO 160
  150 NT=1
      LM(3)=5
      I=1
      K=3
      J=2
      XC=(RR(1)+RR(2)+RR(3))/3.
      YC=(ZZ(1)+ZZ(2)+ZZ(3))/3.
      RR(5)=RR(3)
      ZZ(5)=ZZ(3)
  160 DO 200 NN=1,NT
      LM(1)=2*I-1
      LM(2)=2*J-1
      U(1)=ZZ(J)-ZZ(K)
      U(2)=ZZ(K)-ZZ(I)
      U(3)=ZZ(I)-ZZ(J)
      V(1)=RR(K)-RR(J)
      V(2)=RR(I)-RR(K)
      V(3)=RR(J)-RR(I)
      AREA=(RR(J)*U(2)+RR(I)*U(1)+RR(5)*U(3))/2.
      VOL=VOL+AREA
      COMM=.25/AREA
      XNT=NT
      COM=2./XNT
      COM=COM*COMM
      DO 180 L=1,3
      II=LM(L)
      ST(1,II)=ST(1,II)+U(L)*COM
      ST(2,II+1)=ST(2,II+1)+V(L)*COM
      ST(3,II)=ST(3,II)+V(L)*COM
      ST(3,II+1)=ST(3,II+1)+U(L)*COM
      DO 180 M=1,3
      JJ=LM(M)
      S(II,JJ)=S(II,JJ)+(U(L)*C(1,1)*U(M)+V(L)*C(3,3)*V(M))*COMM
      S(II,JJ+1)=S(II,JJ+1)+(U(L)*C(1,2)*V(M)+V(L)*C(3,3)*U(M))*COMM
      S(II+1,JJ+1)=S(II+1,JJ+1)+(V(L)*C(1,1)*V(M)+U(L)*C(3,3)*U(M))*COMM
      S(JJ+1,II)=S(II,JJ+1)
  180 CONTINUE
      I=J
      J=J+1
  200 CONTINUE
      IF(IX(N,3).EQ.IX(N,4))GO TO 250
      DO240 I=1,2
      KK=10-I
      DO 240 K=1,KK
      CC=S(KK+1,K)/S(KK+1,KK+1)
      DO 230 J=1,3
  230 ST(J,K)=ST(J,K)-CC*ST(J,KK+1)
      DO 240 J=1,KK
  240 S(J,K)=S(J,K)-CC*S(J,KK+1)
  250 CONTINUE
      IF(LLL.EQ.1) GO TO 400
      SIG(1)=-SIGI(N,4)
      SIG(2)=-SIGI(N,5)
      SIG(3)=-SIGI(N,6)
      GO TO 500
C     * * * * * * * * * * * * * * * * * * * * * ** * * * * * * * * * * * *
C     INITIAL STRESSES ADDED  TO THE CALCULATED STRESSES
C     * * * * * * * * * * * * * * * * * * * * * ** * * * * * * * * * * * *
  400 DT=TEMP-Q
      DX=FE(3)*DT
      DY=FE(3)*DT
C
C     INITIAL STRESSES CALCULATED TOTAL HEIGHT =88 FT. AND VALUE OF
C     COEFF. KO =0.2  AND UNIT WEIGHT = 150 FOR ALL MATERIALS
      SIGI(N,2)= -150.0 * (88.0 - YC)
      SIGI(N,1)= 0.2* SIGI(N,2)
      SIGI(N,3)=0.0
C
      SIG(1)=-C(1,1)*DX-C(1,2)*DY+SIGI(N,1)
      SIG(2)=-C(2,1)*DX-C(2,2)*DY+SIGI(N,2)
      SIG(3)=SIGI(N,3)
C
  500 DO 520 I=1,8
      P(I)=0.0
      DO 510 J=1,3
  510 P(I)=P(I)-ST(J,I)*SIG(J)
  520 P(I)=P(I)*VOL
      IF(LLL.EQ.1) GO TO 540
      DO 530 I=1,3
  530 SIG(I)=0.0
      GO TO 600
  540 MM=4
      IF(IX(N,3).EQ.IX(N,4)) MM=3
      XMM=MM
      DY=VOL*ACELZ*RO(MTYPE)/XMM
      DX=VOL*ACELR*RO(MTYPE)/XMM
```

```
ELEM151        DO 550 I=1,MM
ELEM152        P(2*I)=P(2*I)+DY
ELEM153    550 P(2*I-1)=P(2*I-1)+DX
ELEM154    600 RETURN
ELEM155        END
```

```
      SUBROUTINE STRESS(R,Z,UR,UZ,T,PR,SIGI,CODE,CU,A,B,IX,IBC,JBC)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON AA(10000),IA(1500)
      COMMON/ONE/ACELR,ACELZ,TEMP,Q,TOL,VOL,NUMNP,NUMEL,NUMMAT,NUMPC,
     *MTYPE,LLL,N,MBAND,NUMBLK,K2K,NCHECK,NP,NL,NEQ,HED(18),NPC
      COMMON/TWO/C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),XC,YC,
     *EE(3),LM(4),E(5,4,10),RO(10),NTC(10),MTC(10)
      COMMON/THREE/MTOT,NTOT,N11,M4,STOP
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),T(NUMNP),PR(NPC,2)
     *,SIGI(NUMEL,6),CODE(NUMNP),IX(NUMEL,5),IBC(NPC),JBC(NPC)
      DIMENSION CU(NEQ),A(NL,MBAND),B(NEQ)
      FOR = 0.0
      MPRINT=0
      DO 600 M=1,NUMEL
      N=M
      MTYPE=IX(N,5)
      SIGI(N,4)=0.
      SIGI(N,5)=0.
      SIGI(N,6)=0.
      IF(IX(N,3).NE.IX(N,2))GO TO 90
      I=IX(N,1)
      J=IX(N,2)
      XC=(R(I)+R(J))/2.0
      YC=(Z(I)+Z(J))/2.0
      DX=R(J)-R(I)
      DY=Z(J)-Z(I)
      XL=DSQRT(DX**2+DY**2)
      DU=B(2*J-1)-B(2*I-1)
      DV=B(2*J)-B(2*I)
      DL=DV*DY/XL+DU*DX/XL
      SIG(1)=E(1,4,MTYPE)*DL*E(I,2,MTYPE)/XL+SIGI(N,1)*E(1,4,MTYPE)
      IF(SIG(1).GT.0.) GO TO 100
      SIGI(N,1)=SIG(1)
      GO TO 500
  100 SIGI(N,4)=E(1,2,MTYPE)*DL/XL+SIGI(N,1)
      SIGI(N,1)=0.
      GO TO 420
   90 CALL ELEMEN(R,Z,UR,UZ,T,PR,SIGI,CODE,CU,A,B,IX,IBC,JBC)
      MM=4
      IF(IX(N,3).NE.IX(N,4))GO TO 170
      MM=3
  170 DO 180 I=1,3
      RR(I)=0.
      DO 180 J=1,MM
      II=2*J
      JJ=2*IX(N,J)
  180 RR(I)=RR(I)+ST(I,II)*B(JJ)+ST(I,II-1)*B(JJ-1)
      DO 190 I=1,3
      DO 185 J=1,3
  185 SIG(I)=SIG(I)+C(I,J)*RR(J)
  190 CONTINUE
      IF(LLL.EQ.1) GO TO 195
      DO 192 I=1,3
  192 SIG(I)=SIG(I)+SIGI(N,I)
  195 CONTINUE
      CC=(SIG(1)+SIG(2))/2.0
      BB=(SIG(1)-SIG(2))/2.
      CR=DSQRT(BB**2+SIG(3)**2)
      SIG(4)=CC+CR
      SIG(5)=CC-CR
      SIG(6)=0.0
      IF((BB.EQ.0.).AND.(SIG(3).EQ.0.)) GO TO 200
      SIG(6)=28.648*DATAN2(SIG(3),BB)
      DX=0.0
  200 SIGI(N,1)=SIG(1)
      SIGI(N,2)=SIG(2)
      SIGI(N,3)=SIG(3)
      IF((SIG(4).LE.0).OR.(MTC(MTYPE).EQ.0)) GO TO 500
      IF(SIG(5).GE.0.00001) GO TO 370
      EPS=SIG(6)/57.296
      CC=DCOS(EPS)
      SS=DSIN(EPS)
      C2=CC*CC
      S2=SS*SS
      SC=SS*CC
C
      DX=EE(2)*SIG(4)
      SIGI(N,4)= SIG(4)*C2+DX*S2
      SIGI(N,5)= SIG(4)*S2+DX*C2
      SIGI(N,6)=SIG(4)*SC-DX*SC
C
C
      GO TO 400
  370 SIGI(N,4)=SIG(1)
      SIGI(N,5)=SIG(2)
      SIGI(N,6)=SIG(3)
  400 SIGI(N,1)=SIG(1)-SIGI(N,4)
      SIGI(N,2)=SIG(2)-SIGI(N,5)
      SIGI(N,3)=SIG(3)-SIGI(N,6)
  420 DX=SIGI(N,4)**2+SIGI(N,5)**2+SIGI(N,6)**2
      DX=DSQRT(DX)
      IF(DX.LE.FUR) GO TO 450
      IJK=N
      FUR=DX
  450 CONTINUE
  500 IF(MPRINT.NE.0) GO TO 550
      WRITE(6,2000)
      MPRINT=50
  550 MPRINT=MPRINT-1
      WRITE(6,2001)N,XC,YC,(SIG(I),I=1,6),DX
  600 CONTINUE
      WRITE(6,2002)FOR,IJK
      IF(FOR.LE.TOL) NCHECK = 0
      RETURN
 2000 FORMAT (7H1EL.NO.  7X 1HX  7X 1HY 4X 8HX-STRESS 4X 8HY-STRESS 3X
     1 9HXY-STRESS 2X 10HMAX-STRESS 2X 10HMIN-STRESS 7H  ANGLE 2X 17HUNB
     2ALANCED FORCE    )
 2001 FORMAT (I7,2F8.2,1P5E12.4,0P1F7.2,1PE20.4)
 2002 FORMAT(30H0MAXIMUM UNBALANCED FORCE =    E12.5,16H IN ELEMENT NO.
     1 I5)
      END
      SUBROUTINE MODIFY(A,NEQ,MBAND,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NEQ,MBAND)
      DO 250 M=2,MBAND
      K=N-M+1
      IF(K) 235,235,230
  230 A(K,M) = 0.0
  235 K=N+M-1
      IF(NEQ-K) 250,240,240
  240 A(N,M) = 0.0
  250 CONTINUE
      A(N,1)=1.0
      RETURN
      END
      SUBROUTINE LOAD(R,Z,UR,UZ,T,PR,SIGI,CODE,CO,A,B,IX,IBC,JBC)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON AA(10000),IA(1500)
      COMMON/ONE/ACELR,ACELZ,TEMP,Q,TOL,VOL,NUMNP,NUMEL,NUMMAT,NUMPC,
     *MTYPE,LLL,N,MBAND,NUMBLK,KKK,NCHECK,NP,NL,NEQ,HED(18),NPC
      COMMON/TWO/C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),XC,YC,
     *EE(3),LM(4),E(5,4,10),RO(10),NTC(10),MTC(10)
      COMMON/THREE/MTOT,NTOT,N11,M4,STOP
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),T(NUMNP),PR(NPC,2)
     *,SIGI(NUMEL,6),CODE(NUMNP),IX(NUMEL,5),IBC(NPC),JBC(NPC)
      DIMENSION CO(NEQ),A(NL,MBAND),B(NEQ)
      DO 50 N=1,NUMNP
      B(2*N-1)=UR(N)
      B(2*N)=UZ(N)
      UR(N)=0.
      UZ(N)=0.
   50 CONTINUE
      IF((NUMPC.EQ.0).OR.(LLL.GT.1)) GO TO 300
      DO 200 L=1,NUMPC
      I=IBC(L)
      J=JBC(L)
      DR=Z(I)-Z(J)
      DZ=R(J)-R(I)
      PP2=(PR(L,2)+PR(L,1))/6.
      PP1=PP2+PR(L,1)/6.
      PP2=PP2+PR(L,2)/6.
      II=2*I
      JJ=2*J
      B(II-1)=B(II-1)+PP1*DR
      B(II)=B(II)+PP1*DZ
      B(JJ-1)=B(JJ-1)+PP2*DR
      B(JJ)=B(JJ)+PP2*DZ
  200 CONTINUE
  300 DO 700 N=1,NUMEL
      I=IX(N,1)
      J=IX(N,2)
      K=IX(N,3)
      L=IX(N,4)
      MTYPE=IX(N,5)
      IF(SIGI(N,4).NE.0.) GO TO 320
      IF(SIGI(N,5).NE.0.) GO TO 320
      IF(SIGI(N,6).NE.0.) GO TO 320
      IF(LLL.EQ.1) GO TO 320
      GO TO 700
  320 CONTINUE
      IF(LLL.EQ.1) GO TO 330
      IF(MTC(MTYPE).EQ.0) GO TO 700
  330 IF(J.EQ.K) GO TO 500
      CALL ELEMEN(R,Z,UR,UZ,T,PR,SIGI,CODE,CO,A,B,IX,IBC,JBC)
      GO TO 600
```

```
500 CALL ONED(R,Z,UR,UZ,T,PR,SIGI,CODE,CU,A,B,IX,IBC,JBC)
    DX=R(J)-R(I)
    DY=Z(J)-Z(I)
    EP=-SIGI(N,4)/E(1,2,MTYPE)
    DX=DX*EP
    DY=DY*EP
    P(1)=S(1,1)*DX+S(1,2)*DY
    P(2)=S(2,1)*DX+S(2,2)*DY
    P(3)=-P(1)
    P(4)=-P(2)
600 DO 620 II=1,4
620 LM(II)=2*IX(N,II)-1
    DO 650 JJ=1,4
    II=LM(JJ)
    B(II)=B(II)+P(2*JJ-1)
650 B(II+1)=B(II+1)+P(2*JJ)
700 CONTINUE
    DO 750 N=1,NUMNP
    IF(CODE(N).EQ.0.) GO TO 750
    IF((CODE(N).EQ.1.).OR.(CODE(N).EQ.3.)) B(2*N-1)=0.
    IF((CODE(N).EQ.2.).OR.(CODE(N).EQ.3.)) B(2*N)=0.0
750 CONTINUE
800 RETURN
    END
```

```
      SUBROUTINE SYMBAN(NG,A,B,NEQ,MBAND,NUMBLK,KKK)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NG,MBAND),B(NEQ)
      NN=NG/2
C
C
      NL=NN+1
      NH=NN+NN
      REWIND 1
      GO TO (1000,2000),KKK
 1000 REWIND 2
      NB=0
      GO TO 150
  100 NB=NB+1
      DO 125 N=1,NN
      NM=NN+N
      DO 125 M=1,MBAND
      A(N,M)=A(NM,M)
  125 A(NM,M)=0.
C
      IF(NUMBLK-NB) 150,200,150
  150 READ (2)((A(N,M),M=1,MBAND),N=NL,NH)
      IF(NB) 200,100,200
  200 DO 300 N=1,NN
      IF(A(N,1).EQ.0.)GO TO 300
      DO 275 L=2,MBAND
      IF(A(N,L).EQ.0.) GO TO 275
      C=A(N,L)/A(N,1)
      I=N+L-1
      J=0
      DO 250 K=L,MBAND
      J=J+1
  250 A(I,J)=A(I,J)-C*A(N,K)
      A(N,L)=C
  275 CONTINUE
  300 CONTINUE
      WRITE(1) ((A(N,M),M=1,MBAND),N=1,NN)
      IF(NUMBLK.EQ.NB)GO TO 900
      GO TO 100
 2000 NQ=0
      NB=0
C
C
      GO TO 450
  400 NB=NB+1
      DO 425 N=1,NN
      NM=NN+N
      DO 425 M=1,MBAND
      A(N,M)=A(NM,M)
  425 A(NM,M)=0.
```

```
      IF(NUMBLK.EQ.NB)GO TO 500
  450 READ (1) ((A(N,M),M=1,MBAND),N=NL,NH)
      IF(NB.EQ.0)GO TO 400
  500 DO 550 N=1,NN
      J=NQ+N
      DO 540 L=2,MBAND
      I=J+L-1
      IF(NEQ-I) 545,540,540
  540 B(I)=B(I)-A(N,L)*B(J)
  545 IF(A(N,1).EQ.0.) A(N,1)=1.
  550 B(J)=B(J)/A(N,1)
      IF(NUMBLK-NB) 600,650,600
  600 NQ=NQ+NN
      GO TO 400
  650 BACKSPACE 1
  700 DO 750 M=1,NN
      N=NN+1-M
      J=NQ+N
      DO 750 L=2,MBAND
      IF(A(N,L).EQ.0.)GO TO 750
      I=J+L-1
      IF(NEQ.LT.I)GO TO 750
      B(J)=B(J)-A(N,L)*B(I)
  750 CONTINUE
      NB=NB-1
      IF(NB.EQ.0) GO TO 900
      BACKSPACE 1
      DO 800 N=1,NN
      NM=NN+N
      DO 800 M=1,MBAND
      A(NM,M)=A(N,M)
  800 A(N,M)=0.
      READ(1)((A(N,M),M=1,MBAND),N=1,NN)
      BACKSPACE 1
      NQ=NQ-NN
      GO TO 700
  900 RETURN
      END
```

CHAPTER II: ELPL - A Computer Program for Plane Strain Analysis of Stresses, Deformations and Progressive Failure in Elastic-Perfectly Plastic Rock

2.1. Purpose and Capability

This program is applicable to plane strain analysis of stresses, deformations and progressive failure in elastic-perfectly plastie material following generalized Mohr-Coulomb yield criterion, and the incremental theory of plasticity. Arbitrary initial stresses, arbitrary sequence of exeavation and construction, arbitrary history of load application can be simulated. One-dimensional elements with preseribed prestress can be ineluded in the analysis. The computer program is applicable to study of stability of underground or surfaee exeavations, evaluation of alternative schemes for cxeavation, eomparative study of support stresses and deformation.

Theoretical development ineorporated in the program is discussed in Part I-Teehnical Report, of this report.

2.2. Program Organization

The computer program is in Fortran language. Tapes 1 and 2 are used as scratcn files. Tapes 5 and 6 are the input/output files. The listing in section 2.5 uses double preeision for real numbers. The program capacity can be altered by changing the dimension of arrays AA and IA. These correspond to the total locations required in double preeision real arrays and integer arrays respectively. NTOT, MTOT at lines MAIN 28, MAIN 27 are set equal to the dimension of AA and IA.

The program eonsists of the following units.

a. MAIN

In this unit, the control information including maximum number of elements, nodal points, different materials, boundary pressure cards, the number of steps of excavation or construction, the maximum number of elements removed from the system or added to it at any step, is read in. This information is used to organize the dimensions of various arrays in the analysis. This done, the analysis moves on to the next unit.

b. Subroutine INPT

This subroutine is called by the unit MAIN. The first step is to read in material property data, for all the different materials that may participate in the system at any stage. Also nodal point coordinates, loads and code on boundary conditions are read in or generated. Element geometry, initial stresses and element thickness is also read in or generated. The initial stresses may be input or computed within the program. The thickness, if not specified, is assumed to be unity. Maximum bandwidth for the system is calculated and dimensions of blocks for generation and storage of the system stiffness matrix are defined. After defining these controls, the incremental structure is analyzed in steps. For each step the number of nodal points, the number of elements, the number of elements removed or added, if any, the number of boundary pressure cards, the material type of the elements added or changed in material properties, the number of nodal points removed or added to the system in the step considered, the thickness of elements added, are read in. Additional information defines whether a step involves excavation or construction. At any step, several changes can be simulataneously introduced into a structure.

For example, addition of several sets of elements of different materials, changes in material properties of other elements, removal of certain elements can all be introduced simultaneously if desired. For each step the boundary pressures are read in. After all the information is assembled, the solution process is transferred to subroutine SOLVE.

c. Subroutine SOLVE

This subroutine called by INPT is concerned with obtaining the stresses and deformations at a given stage of the incremental structure allowing for progressive failure. To trace the progressive failure, the solution process traces a sequence of elements reaching yield under the loads applied. This sequence of yield is associated with a proportion of the load application and is described in the output as successive approximations with increasing 'stress ratio.' The procedure consists of applying the total load and then scaling it according to the minimum rates of load increment needed to ensure an excursion to yield by one element at a time.

The SOLVE subroutine calls ONED and QUAD to obtain stiffness of one-dimensional and two-dimensional elements respectively. This is added to system stiffness which is stored on tape after modification for prescribed displacement boundary conditions. Solution to the stiffness equations is obtained in subroutine BANSOL and subroutine STRESS defines the stresses, the stress ratio, the scaling of stress increments and the control for continuation of progressive failure analysis.

d. Subroutine ONED

This subroutine generates stiffness of one-dimensional elements as well as the forces corresponding to the unbalanced stress defined by the difference in total load application and the load taken by the system in the current approximation in progressive failure analysis.

c. Subroutine QUAD

This subroutine generates the stiffness for two-dimensional elements (quadrilaterals or triangles). For the current load increment, an element is either elastic or has reached yield and is plastic. The subroutine STRSTR is called to obtain the stress-strain relationship. Gravity loads, and loads corresponding to unbalanced stress are generated in this subroutine.

f. Subroutine STRSTR

This subroutine is called by QUAD and also by STRESS. It defines the stress-strain relationship for the elastic or plastic materials as the case may be. For the elastic case the relationship is

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E^*}{1 - \nu^{*2}} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu^*}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

and $\sigma_z = \nu\,(\sigma_x + \sigma_y)$

where $E^* = \dfrac{E}{1 - \nu^2}$, $\nu^* = \dfrac{\nu}{1 - \nu}$

and E, ν are respectively the Young's modulus and Poisson's ratio for the isotropic elastic material. If anisotropy is to be considered, the above relationship can be modified to reflect that property.

For the plastic domain the stress-strain relation is

$$\begin{Bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{\tau}_{xy} \\ \dot{\sigma}_z \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \\ D_{41} & D_{42} & D_{43} \end{bmatrix} \begin{Bmatrix} \dot{\epsilon}_x \\ \dot{\epsilon}_y \\ \dot{\gamma}_{xy} \end{Bmatrix}$$

where $D_{11} = 2G\,(1 - h_2 - 2h_1\,\sigma_x - h_3\,\sigma_x{}^2)$

$D_{21} = D_{12} = -\,2G\,(+h_2 + h_1\,(\sigma_x + \sigma_y) + h_3\,\sigma_x\,\sigma_y)$

$D_{31} = D_{13} = -\,2G\,(h_1\,\tau_{xy} + h_3\,\sigma_x\,\tau_{xy})$

$D_{22} = 2G\,(1 - h_2 - 2h_1\,\sigma_y - h_3\,\sigma_y{}^2)$

$D_{32} = D_{23} = -\,2G\,(h_1\,\tau_{xy} + h_3\,\sigma_y\,\tau_{xy})$

$D_{33} = 2G\,(.5 - h_3\,\tau_{xy}{}^2)$

$D_{41} = -\,2G\,(h_2 + h_1\,(\sigma_x + \sigma_z) + h_3\,\sigma_x\,\sigma_z)$

$D_{42} = -\,2G\,(h_2 + h_1\,(\sigma_y + \sigma_z) + h_3\,\sigma_y\,\sigma_z)$

$D_{43} = -\,2G\,(h_1\,\tau_{xy} + h_3\,\tau_{xy}\,\sigma_z)$

$$2G = \frac{E}{(1 + \nu)}$$

$$h_1 = \frac{.5\,h_4}{h_5\;J_2^{\frac{1}{2}}}$$

$$h_2 = \frac{h_4\,h_6}{h_5} - \frac{\nu}{(1 - 2\nu)}\;\frac{k}{h_5\;J_2^{\frac{1}{2}}}$$

$$h_3 = \frac{.5}{h_5\;J_2}$$

$$h_4 = 3\alpha \frac{K}{G} - \frac{J_1'}{3 J_2^{\frac{1}{2}}}$$

$$h_5 = 1 + 9\alpha^2 \frac{K}{G}$$

$$h_6 = \frac{J_1'}{J_2^{\frac{1}{2}}}$$

K = bulk modulus = $\dfrac{E}{3\,(1 - 2\nu)}$

$J_1' = \sigma_x + \sigma_y + \sigma_z$ = first invariant of the stress tensor

J_2 = the second invariant of the stress deviation tensor.

g. Subroutine MODIFY

This subroutine is called by the subroutine SOLVE to modify the stiffness matrix for prescribed displacement boundary conditions. The modified matrix is returned to SOLVE.

h. Subroutine BANSOL

This subroutine called by the subroutine SOLVE solves the stiffness equations by gaussian elimination using auxiliary storage files 1 and 2. Results are stored in B array and returned to SOLVE.

i. Subroutine STRESS

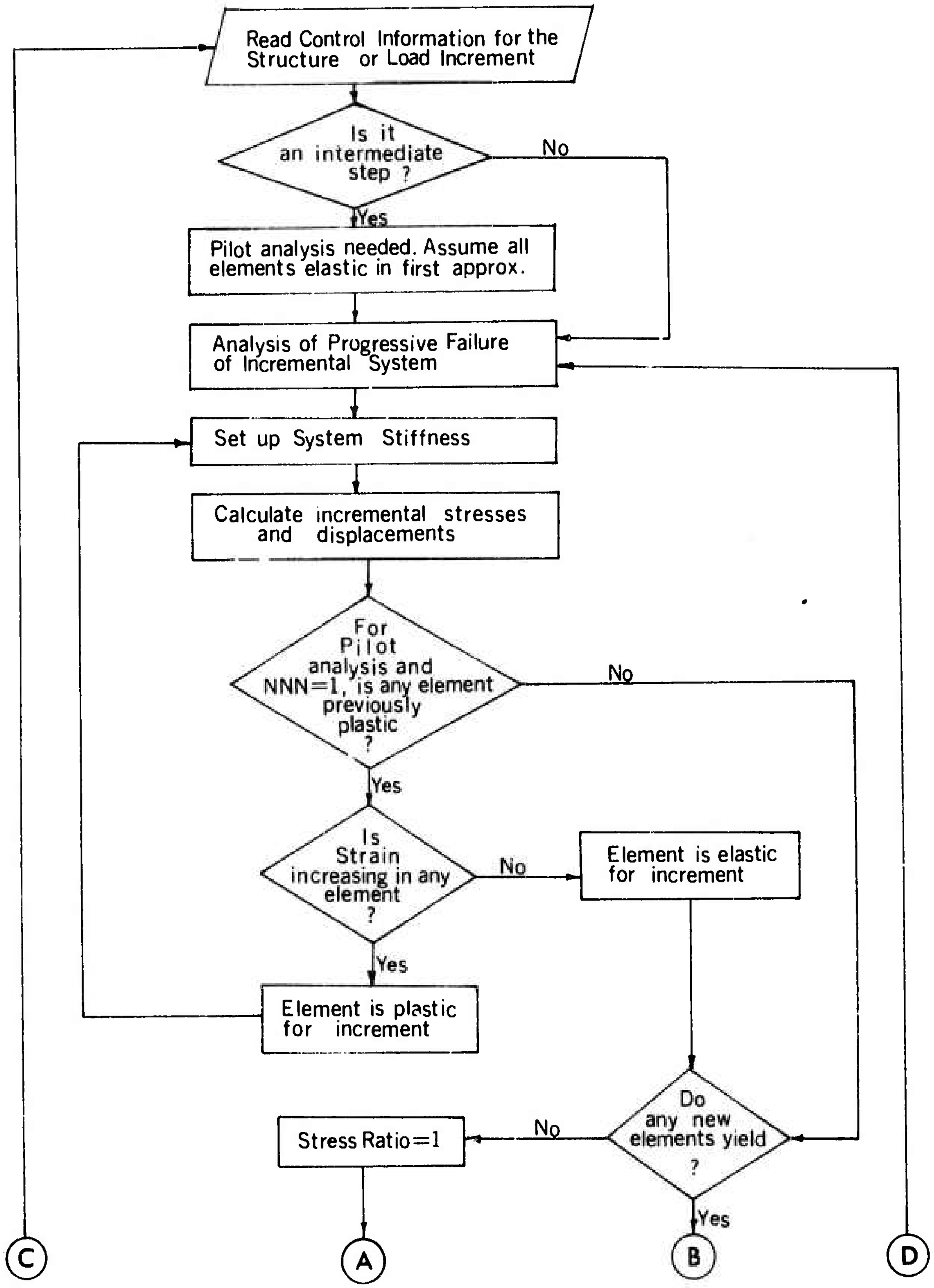
This subroutine is called by the subroutine SOLVE after displacements corresponding to a load increment have been computed. As a first step the entire load is assumed to be applied and the resulting stress state checked for possible excursion beyond yield. If the total load application shows an element passing from the elastic to the plastic stage, the stress ratio is computed as explained in Part I-Technical Report. The minimum stress ratio of all elements corresponds to the least load increment necessary for at least one more element to pass from elastic to the plastic range. The stress ratio also has to be such that elements previously on the yield surface do not depart from this surface by more than the designed tolerance. To avoid unnecessarily lengthy computation, the ratio is kept above a certain minimum. Elements yielding within a small fraction of the load corresponding to the stress ratio are assumed also to have become plastic for the purpose of subsequent computations.

The stiffness for elements in plastic range is stress dependent. Thus over an increment of load the stiffness will change. Noting that plastic yield is, in general, local in character, it is reasonable to assume that the change in stiffness will affect the stress without significantly altering the strains or displacements. An iterative procedure is included to allow for this. Also to determine whether elements initially in plastic range unload elastically in a non-monotonic loading sequence, a pilot analysis is carried out, assuming all elements to be elastic, whenever the structure geometry is changed or a new loading applied. Details of these procedures are discussed in Part I-Technical Report.

After evaluation of stress ratio, the correct stresses in all elements are calculated corresponding to yield of the next group of elements. The difference between the incremental stress corresponding to the total load increment and the stress allowed by the stress ratio is treated as unbalanced stress and applied as psuedo-load in the next iteration.

2.3. Input Data

Input to the program is a sequence of punched cards in the following order and format:


Read Control Information for the Structure or Load Increment
Is it an intermediate step ?
No
Yes
Pilot analysis needed. Assume all elements elastic in first approx.
Analysis of Progressive Failure of Incremental System
Set up System Stiffness
Calculate incremental stresses and displacements
For Pilot analysis and NNN=1, is any element previously plastic ?
No
Yes
Is Strain increasing in any element ?
No
Element is elastic for increment
Yes
Element is plastic for increment
Do any new elements yield ?
No
Stress Ratio=1
Yes
C
A
B
D

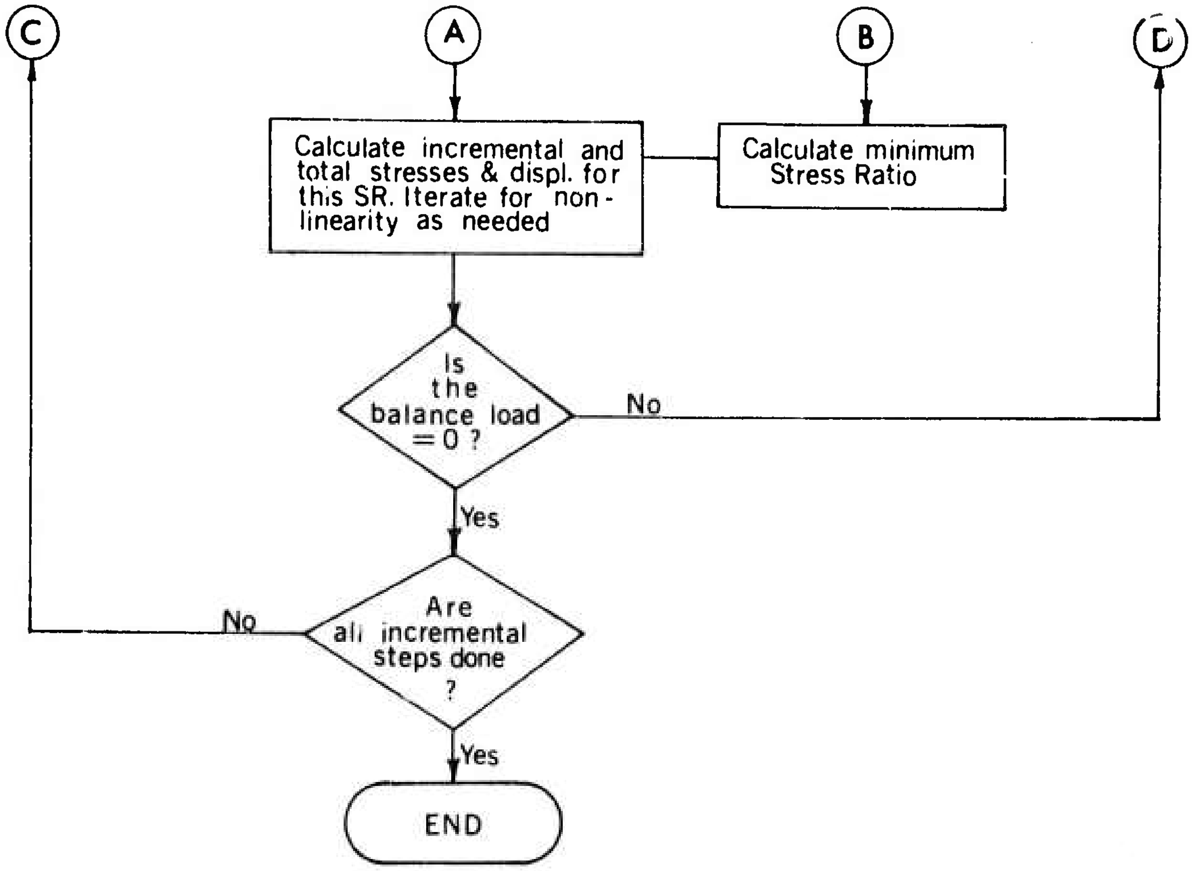

C
A
B
D
Calculate incremental and total stresses & displ. for this SR. Iterate for non-linearity as needed
Calculate minimum Stress Ratio
Is the balance load =0 ?
No
Yes
Are all incremental steps done ?
No
Yes
END

FIG. II-1. Flow Chart for Progressive Failure of Elastic-Plastic Incremental System

a. First Card. Job Title (18A4)

This card carries a descriptive title to identify the job.

b. Second Card. Control Information (4I5, 2F10.2, 3I5)

This card carries the control information for the problem being solved.

Information	Columns
Total number of nodal points (NUMNP)	1-5
Total number of elements (NUMEL)	6-10
Number of different materials (NUMMAT)	11-15
Number of pressure cards (NUMPC)	16-20
Acceleration in X-direction (ACELR)	21-30
Acceleration in Y-direction (ACELZ)	31-45
Total number of excavation and construction steps (NSTEP)	46-50
Code to designate whether initial stress is being fed as data or will be evaluated before the first step (MCASE) MCASE = 0, initial stress will be evaluated = 1, initial stress is fed as data	51-55
Maximum number of elements/nodal points removed or added in any one step (NMR).	56-60

c. Material property cards

One set of cards will be provided for each material. Each set will consist of the following cards:

i. First Card (I5, F10.0)

Information	Columns
Material identification number (MTYPE)	1-5
Mass density of material (RO)	6-15

ii. Seeond Card (5F10.0) for two-dimensional element

Information	Columns
Elastic modulus	1-10
Poisson's ratio	11-20
Cohesion	21-30
Angle of internal frietion in degrees	31-40

or

iii. Seeond card (5F10.0) for one-dimensional element

Information	Columns
Elastie modulus	1-10
Poisson's ratio	11-20
Code = 1, if element is prestressed = 0, if element is not prestressed	21-30
Allowable eompressive strength of the material if the element has prestressing (eode = 1)	31-40
Area of one-dimensional element	41-50

d. Nodal Point Data (I5, F5.0, 5 F10.0)

One card for each nodal point with the following information is provided.

Information	Columns
Nodal point number	1-5
Type of nodal point (CODE)	6-10
X-ordinate	11-20

Y-ordinate	21-30
X load or displacement (XR)	31-40
Y load or displacement (XZ)	41-50

The code will be defined as follows:

CODE	Implication
0.0	XR is the specified X-load and XZ is the specified Y-load
1.0	XR is the specified X-displacement and XZ is the specified Y-load
2.0	XR is the specified X-load and XZ is the specified Y-displacement
3.0	XR is the specified X-displacement and XZ is the specified Y-displacement.

Nodal point cards must be in numerical sequence. Nodal points for which no cards are input will be generated by interpolation between specified nodal points. These points will have CODE and loads set equal to zero. The X, Y coordinates will be linearly interpolated.

e. Element Data (6I5, 5F10.0)

One card for each element, in numerical sequence, will show:

Information	Columns
Element number	1-5
Nodal point I	6-10
Nodal point J	11-15
Nodal point K	16-20
Nodal point L	21-25
Material type	26-30

Nodal points are labelled I, J, K, L counter-clockwise.

Initial stresses:

Component in X-direetion	31-40
Component in Y-direetion	41-50
Shearing stress in X-Y plane	51-60
Component in Z-direction	61-70
Thiekness of element (TH)	71-80

If the eolumns for thiekness are left blank, thickness will be taken as 1.0. Elements omitted from the sequenee will be generated. The material type and thickness for generated elements is the same as for the preeeding element.

f. If the initial stresses are to be evaluated (i.e. if MCASE = 0 in eard b), the following card should be included; otherwise proceed to g.

Initial Stress Evaluation

i. First eard (18A4)

This eard gives the deseriptive title of thc step.

ii. Second card (4I5)

Information	Columns
Number of nodal points in this step	1-5
Number of elements in this step	6-10
Blank	11-15
Number of pressure boundary eards	16-20

g. Incremental Step Information

One set of cards will be provided for each step of construction or dismantling as follows:

i. First card (18A4)

This card will give a descriptive title of the step.

ii. Second card (10I5)

Information	Columns
Number of nodal points in this step	1-5
Number of elements in this step	6-10
Number of elements removed/added	11-15
Number of pressure boundary cards	16-20
Material type of new elements	21-25
Code = 0 for dismantling = 1 for construction	26-30
Number of nodal points removed/added	31-35

If in addition to adding/removing elements, it is desired to change the material type of some existing elements, use columns 36-45.

Number of elements for which material type is to be changed	36-40
New material type of changed elements	41-45

If it is desired to add/remove two material types in one step, use columns 46-50.

KMORE = 1, another material is being added/removed = 0, no other material is being added/removed.	46-50

iii. Elements removed/added (16I5)

One or more cards will indicate the element numbers removed or added in this step. Total number of elements should be the same as in columns 11-15 in card ii.

iv. Nodal points removed/added (16I5)

One or more cards will indicate the nodal point numbers removed or added in this step. The total number should be the same as in columns 31-35 of card ii.

v. Elements for which material type changed (16I5)

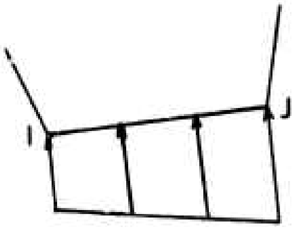
One or more cards will indicate the element numbers for which material type has been changed. Total number should be the same as in columns 36-40 in card ii.

If KMORE = 1, repeat card i to v for the next material; otherwise proceed to card vi.

vi. Pressure boundary cards (2I5, 2F10.0)

The cards representing pressure at the boundary will be as follows:

Information	Columns
Nodal point I	1-5
Nodal point J	6-10
Total normal pressure at I	11-20
Total normal pressure at J	21-30



As shown in the sketch, the boundary of the element must be on the left hand side as one progresses from I to J. Surface tensile force is input as a negative pressure.

2.4. Output information

The following information is developed and printed by the program:

a. Reprint of the input data

b. Cumulative nodal point displacement after each step

c. Stresses at the center of each element after each step

d. Stress ratios for elements which can yield with full application of the balance load

e. The strss ratio applicable for any iteration and the consideration leading to its determination c.g. yielding of next element, minimum stress ratio, maximum stress ratio, ensuring that elements previously in yield surface do not move away from that surface beyond a specified tolerance.

f. The proportion of total load increment applied upto the end of current iteration.

g. The next group of elements reaching yield

h. The failure ratio for each element.

2.5. Fortran Listing

```
C     *   FINITE ELEMENT ANALYSIS OF ELASTIC-PERFECTLY PLASTIC      *
C     *   MOHR-COULOMB SULIDS UNDER PLANE STRAIN.  INCREMENTAL       *
C     *   CONSTRUCTION  EXCAVATION AND INITIAL STRESSES ARE          *
C     *   CONSIDERED.   A 4-CST QUADRILATERAL ELEMENT IS USED.       *
C     *   THE FORMULATION IS DOCUMENTED IN THE FINAL REPORT          *
C     *   ON CONTRACT H0210017 BETWEEN THE OHIO STATE UNIVERSITY     *
C     *   AND THE UNITED STATES BUREAU OF MINES SUPPORTED BY         *
C     *   THE ADVANCED RESEARCH PROJECTS AGENCY.                     *
C     *   PROGAMMERS:   R.D.SINGH AND K.J.SINGH                      *
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TOTAL/ AA(30000), IA(7000)
      COMMON/ONE/NUMNP,NUMEL,NUMMAT,NUMPC,NPMAX,NELMAX,N,NNN,KKK,KLK,
     .    ISHIFT,MBAND,NBAND,NUMBLK,MTYPE,NCASE,MCASE,NMR,NL,NPP,NEQ,NPC
     .    ,NP,NSTEP,NCODE
      COMMON/TWO/RR(5),ZZ(5),S(10,10),P(10),ST(3,10),C(4,4),SIG(7),FF(4)
     .    ,TITLE(18),SR1,SR2,XC,YC,ACELR,ACELZ,BCELR,BCELZ,VOL
      COMMON/THREE/STOP,MTOT,NTOT,N13,N14,M8
C
C         ACELR,ACELZ ARE BODY FORCE INTENSITIES.   MCASE IS INPUT AS
C         ZERO IF INITIAL STRESSES HAVE TO BE COMPUTED AS PART OF
C         THE ANALYSIS.
C
      READ(5,1000)TITLE ,NUMNP,NUMEL,NUMMAT,NUMPC,ACELR,ACELZ,
     .              NP,NSTEP,MCASE,NMR
      WRITE (6,2000) TITLE,NUMNP,NUMEL,NUMMAT,NUMPC,ACELR,ACELZ,NP,
     .               NSTEP,NMR
      MTOT=7000
      NTOT=30000
      NPC=NUMPC
      IF(NUMPC.EQ.0) NPC=1
      IF(NMR.EQ.0) NMR=1
      N1=1
      N2=N1+NUMNP
      N3=N2+NUMNP
      N4=N3+NUMNP
      N5=N4+NUMNP
      N6=N5+NUMNP
      N7=N6+2*NUMNP
      N8=N7+5*NUMMAT
      N9=N8+NUMMAT
      N10=N9+2*NPC
      N11=N10+8*NUMEL
      N12=N11+NUMEL
      N13=N12+NUMEL
      M1=1
      M2=M1+5*NUMEL
      M3=M2+NUMEL
      M4=M3+NPC
      M5=M4+NPC
      M6=M5+NMR
      M7=M6+NMR
      M8=M7+NMR
      JJ=M8-MTOT
      IF(JJ.LE.0) GO TO 100
      WRITE(6,3000)JJ
      CALL EXIT
  100 CONTINUE
      NEQ=2*NUMNP
      CALL INPT  (AA(N1),AA(N2),AA(N3),AA(N4),AA(N5),AA(N6),AA(N7),
     *            AA(N8),AA(N9),AA(N10),AA(N11),AA(N12),  IA(M1),IA(M2),
     *            IA(M3),IA(M4),IA(M5),IA(M6),IA(M7)  )
 1000 FORMAT(18A4/4I5,2F10.2,4I5)
 2000 FORMAT(1H1,18A4/
     .30H0  NUMBER OF NODAL POINTS                                  I10/
     .30H0  NUMBER OF ELEMENTS                                      I10/
     .30H0  NUMBER OF DIFF. MATERIALS                               I10/
     .30H0  NUMBER OF PRESSURE CARDS                                I10/
     .29H0  X-ACCELERATION                                        F11.1/
     .29H0  Y-ACCELERATION                                        F11.1/
     .30H0  NUMBER OF APPROXIMATIONS                                I10/
     .30H0  NUMBER OF STEPS                                         I10/
     .69H0  MAX. NO. OF NODAL POINTS OR ELEMENTS ADDED OR REMOVED AT ANY
     . STEP            I5)
 3000 FORMAT(70H  EXECUTION TERMINATED IN MAIN PROGRAM. REQUIRED CORE EX
     .CEEDS MTOT BY                          I10)
      STOP
      END
      SUBROUTINE INPT (R,Z,UR,UZ,CODE,BO,E,RO,PR,SIGI,TH,ER,IX,MTAG,
     *             IBC,JBC,NUMR,NUMR1,NNP)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TOTAL/ AA(30000), IA(7000)
      COMMON/ONE/NUMNP,NUMEL,NUMMAT,NUMPC,NPMAX,NELMAX,N,NNN,KKK,KLK,
     .    ISHIFT,MBAND,NBAND,NUMBLK,MTYPE,NCASE,MCASE,NMR,NL,NPP,NEQ,NPC
     .    ,NP,NSTEP,NCODE
      COMMON/TWO/RR(5),ZZ(5),S(10,10),P(10),ST(3,10),C(4,4),SIG(7),EE(4)
     .    ,TITLE(18),SR1,SR2,XC,YC,ACELR,ACELZ,BCELR,BCELZ,VOL
      COMMON/THREE/STOP,MTOT,NTOT,N13,N14,MB
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),CODE(NUMNP),
     .    BO(NEQ),E(5,NUMMAT),RO(NUMMAT),PR(NPC   ,2),SIGI(NUMEL,8),
     .    TH(NUMEL),ER(NUMEL),IX(NUMEL,5),MTAG(NUMEL),IBC(NPC),
     .    JBC(NPC   ),NUMR(NMR),NUMR1(NMR),NNP(NMR)
C
C         INPUT MATERIAL PROPERTIES
C
      DO 55 M=1,NUMMAT
      READ (5,1001) MTYPE,RO(MTYPE)
      WRITE(6,2001) MTYPE,RO(MTYPE)
      READ (5,1002) (E(J,MTYPE),J=1,5)
      WRITE(6,2002) (E(J,MTYPE),J=1,5)
   55 CONTINUE
C
C         READ AND PRINT NODAL POINT DATA
C
      WRITE (6,2003)
      L=0
   60 READ  (5,1003) N,CODE(N),R(N),Z(N),UR(N),UZ(N)
      NL=L+1
      IF (N.EQ.1) GO TO 70
      ZX=N-L
      DR=(R(N)-R(L))/ZX
      DZ=(Z(N)-Z(L))/ZX
   70 L=L+1
      IF(N-L) 100,90,80
   80 CODE(L)=0.0
      R(L)=R(L-1)+DR
      Z(L)=Z(L-1)+DZ
      UR(L)=0.0
      UZ(L)=0.0
      GO TO 70
   90 IF(NUMNP-N) 100,110,60
  100 WRITE (6,2005) N
      CALL EXIT
  110 WRITE (6,2004)((K,CODE(K),R(K),Z(K),UR(K),UZ(K)),K=1,NUMNP)
C
C         READ AND PRINT ELEMENT DATA
C
      WRITE (6,2006)
      N=0
  130 READ (5,1004) M,(IX(M,I),I=1,5),(SIGI(M,I),I=1,4),TH(M)
      IF (TH(M).EQ.0.) TH(M)=1.0
      ZX=M-N
      IF (N.EQ.0) GO TO 140
      DO 135 I=1,4
  135 SIG(I)=(SIGI(M,I)-SIGI(N,I))/ZX
  140 N=N+1
      IF(M.LE.N) GO TO 170
      IX(N,1)=IX(N-1,1)+1
      IX(N,2)=IX(N-1,2)+1
      IX(N,3)=IX(N-1,3)+1
      IX(N,4)=IX(N-1,4)+1
      IX(N,5)=IX(N-1,5)
      DO 160 I=1,4
  160 SIGI(N,I)=SIGI(N-1,I)+SIG(I)
      TH(N)=TH(N-1)
  170 IF(M.GT.N) GO TO 140
      IF(NUMEL.GT.N) GO TO 130
      WRITE(6,2007) ((N,(IX(N,I),I=1,5),(SIGI(N,I),I=1,4),TH(N)),N=1,NUM
     .                 EL)
C
C        INITIALIZATION OF UNBALANCED STRESSES AND CUMULATIVE
C        DISPLACEMENTS
C
      DO 195 N=1,NUMEL
      MTAG(N)=0
      SIGI(N,5)=0.
      SIGI(N,6)=0.
      SIGI(N,7)=0.
  195 SIGI(N,8)=0.
      DO 200 N=1,NUMNP
      BO(2*N-1)=0.
  200 BO(2*N)=0.
C
C        CALCULATE MAXIMUM BAND-WIDTH FOR THE INCREMENTAL SYSTEM
C
      J=0
      DO 250 N=1,NUMEL
      DO 250 I=1,4
      DO 250 L=1,4
      KK=IABS(IX(N,I)-IX(N,L))
      IF(KK.GT.J) J=KK
  250 CONTINUE
      NBAND=2*J+2
      WRITE(6,3000) NBAND
C
C        CALCULATE BLOCKSIZE CONSISTENT WITH AVAILABLE CORE
C
      NL=(NTOT-N13+1)/(NBAND+1)
      NLL=NEQ+3
      IF(NL.GT.NLL) NL=NLL
      NL=NL/4
      NL=4*NL
      N14=N13+NL
      NB=2*NBAND
      NZ=N13+NB+NB*NBAND-1
      WRITE (6,4010) NZ
      IF(NZ.GT.NTOT) CALL EXIT
      IF (NL.LT.NB) CALL EXIT
C     ...........................................................
C     .  READ CONTROL INFORMATION FOR THE NEXT STEP IN INCREMENTAL  .
C     .  ANALYSIS                                                   .
C     ...........................................................
      NCASE=0
      IF(MCASE.NE.0) NCASE=1
      IF(MCASE.NE.0) MCASE=1
  300 READ(5,1008) TITLE
      WRITE(6,2011) TITLE
      READ(5,1009) NPMAX,NELMAX,NUMER,NUMPC,MTYPE,NCODE,NPMIS,
     .                   NUMER1,MTYPE1,KMORE,THICK
      IF(THICK.EQ.0.) THICK=1.0
      IF(NUMER.EQ.0) GO TO 310
      READ(5,1007)  (NUMR(N), N= 1,NUMER)
      DO 305  I=1,NUMER
      NUM=NUMR(I)
      TH(NUM)=THICK
  305 CONTINUE
  310 IF(NPMIS.EQ.0) GO TO 320
      READ(5,1007)  (NNP(M), M = 1,NPMIS)
  320 IF (NUMER1.EQ.0) GO TO 330
      READ(5,1007) ( NUMR1(N),N=1,NUMER1)
C
C        DETERMINE BAND-WIDTH FOR THIS STEP
C
  330 J=0
      IF(KMORE.NE.0) GO TO 342
      DO 340 N = 1,NELMAX
      DO 340 I=1,4
      DO 340 L=1,4
      KK=IABS(IX(N,I)-IX(N,L))
      IF(KK.GT.J) J=KK
  340 CONTINUE
      MBAND=2*J+2
  342 IF(NPMIS.EQ.0) GO TO 346
      DO 345 I = 1, NPMIS
      J=NNP(I)
      BO(2*J-1)=0.
      BO(2*J)=0.
  345 CONTINUE
```

```
C
C        PRINT CONTROL INFORMATION FOR THE CURRENT STEP IN INCREMENTAL
C        ANALYSIS
C
  346 IF(NCODE.EQ.1) GO TO 410
      IF(NUMER.EQ.0) GO TO 351
      WRITE(6,2012)NCASE,NPMAX,NELMAX,NUMPC,NPMIS,MTYPE,
     .              (NUMR(N),N=1,NUMER)
      GO TO 352
  351 WRITE(6,2012)NCASE,NPMAX,NELMAX,NUMPC,NPMIS,MTYPE
  352 IF(NPMIS.EQ.0)  GO TO 420
      WRITE(6,2014) (NNP(M) ,M = 1,NPMIS)
      GO TO 420
  410 IF(NUMER.EQ.0) GO TO 411
      WRITE(6,2013)NCASE,NPMAX,NELMAX,NUMPC,NPMIS,MTYPE,
     .              (NUMR(N),N=1,NUMER)
      GO TO 412
  411 WRITE(6,2013)NCASE,NPMAX,NELMAX,NUMPC,NPMIS,MTYPE
  412 IF(NPMIS.EQ.0)  GO TO 420
      WRITE(6,2014) (NNP(M) ,M = 1,NPMIS)
  420 CONTINUE
      IF (NUMER1.EQ.0) GO TO 440
      WRITE (6,2018) (NUMR1(N),N=1,NUMER1)
      WRITE (6,2020) MTYPE1
  440 IF (KMORE.NE.0) GO TO 510
      WRITE (6,2025) MBAND
C
C        READ AND PRINT BOUNDARY PRESSURES, IF ANY
C
      IF(NUMPC.EQ.0) GO TO 510
      WRITE (6,2008)
      DO 500 L=1,NUMPC
      READ(5,1005) IBC(L),JBC(L),PR(L,1),PR(L,2)
  500 WRITE(6,2009)IBC(L),JBC(L),PR(L,1),PR(L,2)
  510 CONTINUE
C
C        FOR ELEMENTS REMOVED OR NEWLY ADDED IN THIS STEP
C        SET INITIAL STRESSES EQUAL TO ZERO
C
      IF (NUMER.EQ.0) GO TO 601
      DO 600 I=1,NUMER
      NUM = NUMR(I)
      IX(NUM,5)=MTYPE
      SIGI(NUM,1)=0.0
      SIGI(NUM,2)=0.0
      SIGI(NUM,3)=0.0
      SIGI(NUM,4)=0.0
      MTAG(NUM)=0
  600 CONTINUE
  601 IF (NUMER1.EQ.0) GO TO 610
```

```
INPT201       DO 606 I=1,NUMERI
INPT202       NUM=NUMR1(I)
INPT203   606 IX(NUM,5)=MTYPE1
INPT204   610 IF (KMORE.NE.0) GO TO 300
INPT205 C
INPT206 C          EVERY ELEMENT IS ASSUMED ELASTIC AT THE START OF EACH STEP
INPT207 C
INPT208       SR1=1.0
INPT209       SR2=0.0
INPT210       NPP=NP
INPT211       IF (NCASE.LT.I) NPP=I
INPT212       CALL SOLVE(P,Z,UP,UZ,CODE,BO,E,RO,PR,SIGI,TH,FR,AA(NI3),AA(NI4),
INPT213      *               IX,MTAG,IBC,JBC,NUMR,NUMR1,NNP)
INPT214       IF(NCASE.LE.NSTEP) GO TO 300
INPT215  1001 FORMAT (115,1F10.0)
INPT216  1002 FORMAT (6F10.0)
INPT217  1006 FORMAT(A6)
INPT218  1003 FORMAT (15,F5.0,4F10.0)
INPT219  1004 FORMAT(615,5F10.0)
INPT220  1005 FORMAT (215,2F10.0)
INPT221  1007 FORMAT(1615)
INPT222  1008 FORMAT(18A4)
INPT223  1009 FORMAT(1015,F10.0)
INPT224  2001 FORMAT (17HOMATERIAL NUMBER= 13, 15H, MASS DENSITY= F12.4)
INPT225  2002 FORMAT(16HOELASTIC MODULUS 14X 2HNU 8X 8HCOHESION 2X 14HFRICTION A
INPT226      .NGLE 2X 13HAREA FOR ONED /(2E16.5,3F16.5))
INPT227  2003 FORMAT (111HINODAL POINT         TYPE  X ORDINATE  Y ORDINATE  X LO
INPT228      .AD OR DISPLACEMENT  Y LOAD OR DISPLACEMENT  PORE PRESSURE )
INPT229  2004 FORMAT (112,F12.2,2F12.3,2E24.7)
INPT230  2005 FORMAT (26HONODAL POINT CARD ERROR N= 15)
INPT231  2006 FORMAT(109HIELEMENT NO.     I     J     K     L    MATERIAL    X-S
INPT232      .TRESS    Y-STRESS   XY-STRESS    Z-STRESS   THICKNESS         )
INPT233  2007 FORMAT(112,416,1112,5F12.3)
INPT234  2008 FORMAT (29HOPRESSURE BOUNDARY CONDITIONS/ 40H     I     J    PRESS
INPT235      .URE I     PRESSURE J                                            )
INPT236  2009 FORMAT (216,2F12.3)
INPT237  2011 FORMAT(1H1 18A4////)
INPT238  2012 FORMAT( 51HO RESULTS AFTER STEP NO.-------------------------   15/
INPT239      . 58HO TOTAL NUMBER OF NODAL POINTS IN SYSTEM AT THIS STEP----  15/
INPT240      . 58HO TOTAL NUMBER OF ELEMENTS IN SYSTEM AT THIS STEP--------  15/
INPT241      . 58HO TOTAL NUMBER OF PRESSURE CARDS AT THIS STEP------------  15/
INPT242      . 58HO TOTAL NUMBER OF NODAL POINTS MISSING AT THIS STEP------  15/
INPT243      . 58HO MATERIAL TYPE OF ELEMENTS ADDED/REMOVED IN THIS STEP---  15/
INPT244      . 58HO ELEMENTS REMOVED IN THIS STEP ARE  -------------------    /
INPT245      . (2015))
INPT246  2013 FORMAT( 51HO RESULTS AFTER STEP NO.-------------------------   15/
INPT247      . 58HO TOTAL NUMBER OF NODAL POINTS IN SYSTEM AT THIS STEP----  15/
INPT248      . 58HO TOTAL NUMBER OF ELEMENTS IN SYSTEM AT THIS STEP--------  15/
INPT249      . 58HO TOTAL NUMBER OF PRESSURE CARDS AT THIS STEP------------  15/
INPT250      . 56HO TOTAL NUMBER OF NODAL POINTS MISSING AT THIS STEP------  15/
INPT251      . 58H0 MATERIAL TYPE BEING PUT IN THIS STEP ------------------  I5/
INPT252      . 58H0 ELEMENTS ADDED   IN THIS STEP ARE  --------------------    /
INPT253      . (20I5))
INPT254  2014 FORMAT( 53H0 NODAL POINTS REMOVED OR ADDED IN THIS STEP ARE-----
INPT255      ./(20I5))
INPT256  2016 FORMAT(I8,2F8.2,1P4E12.4,0P1F8.5)
INPT257  2017 FORMAT(I10,2F20.7)
INPT258  2018 FORMAT (52H0 ELEMENTS FOR WHICH MATERIAL HAS BEEN CHANGED ARE--
INPT259      ./(20I5))
INPT260  2020 FORMAT (54H0 NEW MATERIAL TYPE OF THE ABOVE CHANGED ELEMENTS IS--
INPT261      . I5)
INPT262  2025 FORMAT(27H BAND WIDTH FOR THIS STEP          I5)
INPT263  3000 FORMAT(//12H BAND WIDTH                I5)
INPT264  4010 FORMAT( 65H THE MINIMUM DIMENSION OF AA AND NTOT NEEDED FOR THIS P
INPT265      .ROBLEM IS                 I10)
INPT266       RETURN
INPT267       END
```

```
      SUBROUTINE SOLVE (R,Z,UR,UZ,CODE,BO,E,RO,PR,SIGI,TH,FR,B,A,IX,
     *                   MTAG,IBC,JBC,NUMR,NUMR1,NNP)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TOTAL/ AA(30000), IA(7000)
      COMMON/ONE/NUMNP,NUMEL,NUMMAT,NUMPC,NPMAX,NELMAX,N,NNN,KKK,KLK,
     .    ISHIFT,MBAND,NBAND,NUMBLK,MTYPE,NCASE,MCASE,NMR,NL,NPP,NEQ,NPC
     .    ,NP,NSTEP,NCODE
      COMMON/TWO/RR(5),ZZ(5),S(10,10),P(10),ST(3,10),C(4,4),SIG(7),FF(4)
     .    ,TITLE(18),SR1,SR2,XC,YC,ACELR,ACELZ,BCELR,BCELZ,VOL
      COMMON/THREE/STOP,MTOT,NTOT,N13,N14,MR
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),CODE(NUMNP),
     .    BO(NEQ),E(5,NUMMAT),RO(NUMMAT),PR(NPC   ,2),SIGI(NUMEL,8),
     .    TH(NUMEL),FR(NUMEL),IX(NUMEL,5),MTAG(NUMEL),IBC(NPC),
     .    JBC(NPC   ),NUMR(NMR),NUMR1(NMR),NNP(NMR)
      DIMENSION B(NL),A(NL,NBAND)
      DIMENSION LM(4)
C
      DO 750 NNN=1,NPP
      KKK=0
      REWIND 2
      ND=NL/2
      NB=ND/2
      STOP=0.0
      NUMBLK=0
      DO 50 N=1,NL
      B(N)=0.0
      DO 50 M=1,MBAND
   50 A(N,M)=0.0
      IF(NCASE.GT.MCASE) GO TO  51
      IF (NNN.GT.1) GO TO  51
      BCELR =ACELR
      BCELZ =ACELZ
   51 CONTINUE
      IF (NNN.GT.1) GO TO  52
      ACELR =BCELR
      ACELZ =BCELZ
      GO TO  55
   52 ACELR =0.0
      ACELZ =0.0
      NUMPC =0
   55 CONTINUE
   60 NUMBLK=NUMBLK+1
      NH=NB*(NUMBLK+1)
      NM=NH-NB
      NS=NM-NB+1
      KSHIFT=2*NS-2
      DO 210 N=1,NELMAX
      IF(IX(N,5).LE.0) GO TO 210
      MTYPE=IX(N,5)
      IF (RO(MTYPE).EQ.0.) GO TO 210
      DO 80 I=1,4
      IF(IX(N,I).LT.NS) GO TO 80
      IF(IX(N,I).LE.NM) GO TO 90
   80 CONTINUE
      GO TO 210
   90 IF(IX(N,3).NE.IX(N,2)) GO TO 95
      CALL ONED (R,Z,UR,UZ,CODE,BO,E,RO,PR,SIGI,TH,FR,IX,MTAG,
     *              IBC,JBC,NUMP,NUMP1,NNP)
      IX(N,5)=-IX(N,5)
      MM=2
      GO TO 130
   95 CALL QUAD (R,Z,UR,UZ,CODE,BO,E,RO,PR,SIGI,TH,FR,IX,MTAG,
     *              IBC,JBC,NUMP,NUMP1,NNP)
      IX(N,5)=-IX(N,5)
      IF (VOL.GT.0.) GO TO 110
      WRITE(6,2000) N
      STOP=1.0
  110 MM=4
      IF (IX(N,3).EQ.IX(N,4)) MM=3
  130 DO 140 I=1,MM
  140 LM(I)=2*IX(N,I)-2
      DO 200 I=1,MM
      DO 200 K=1,2
      II=LM(I)+K-KSHIFT
      KK=2*I-2+K
      B(II)=B(II)+P(KK)
      DO 200 J=1,MM
      DO 200 L=1,2
      JJ=LM(J)+L-II+1-KSHIFT
      LL=2*J-2+L
      IF(JJ.LE.0) GO TO 200
      A(II,JJ)=A(II,JJ)+S(KK,LL)*TH(N)
  200 CONTINUE
  210 CONTINUE
      DO 220 N=NS,NM
      IF (N.GT.NPMAX) GO TO 220
      K=2*N-KSHIFT
      B(K)=B(K)+UZ(N)
      B(K-1)=B(K-1)+UR(N)
  220 CONTINUE
C
      IF (NUMPC.EQ.0) GO TO 310
      DO 300 L=1,NUMPC
      I=IBC(L)
      J=JBC(L)
      DR=Z(I)-Z(J)
      DZ=R(J)-R(I)
      PP2=(PR(L,2)+PR(L,1))/6.
      PP1=PP2+PR(L,1)/6.
      PP2=PP2+PR(L,2)/6.
      II=2*I-KSHIFT
      JJ=2*J-KSHIFT
      IF((II.LE.0).OR.(II.GT.ND)) GO TO 265
      B(II-1)=B(II-1)+PP1*DR
      B(II)=B(II)+PP1*DZ
  265 IF((JJ.LE.0).OR.(JJ.GT.ND)) GO TO 300
      B(JJ-1)=B(JJ-1)+PP2*DR
      B(JJ)=B(JJ)+PP2*DZ
  300 CONTINUE
  310 DO 400 M=NS,NH
      IF(M.GT.NPMAX) GO TO 400
      U=UR(M)
      N=2*M-1-KSHIFT
      IF (CODE(M)) 390,400,316
  316 IF(CODE(M).EQ.1.) GO TO 370
      IF(CODE(M).EQ.2.) GO TO 390
      IF (CODE(M)-3.) 390,380,390
  370 CALL MODIFY(A,B,NL ,MBAND,NBAND,N,U)
      GO TO 400
  380 CALL MODIFY(A,B,NL ,MBAND,NBAND,N,U)
  390 U=UZ(M)
      N=N+1
      CALL MODIFY(A,B,NL ,MBAND,NBAND,N,U)
  400 CONTINUE
      WRITE (2) (B(N),(A(N,M),M=1,MBAND),N=1,ND)
      DO 420 N=1,ND
      K=N+ND
      B(N)=B(K)
      B(K)=0.0
      DO 420 M=1,MBAND
      A(N,M)=A(K,M)
  420 A(K,M)=0.0
      IF(NM.LT.NPMAX) GO TO 60
      IF (STOP.NE.0.) CALL EXIT
      DO 600 N=1,NUMNP
      UZ(N)=0.
  600 UR(N)=0.
      N15=N14+2*NUMNP
      N16=N15+4*NUMEL
      N17=N16+NUMEL
      CALL BANSOL (NL ,NBAND,MBAND,NUMBLK,B,A)
      CALL STRESS (R,Z,UR,UZ,CODE,BO,E,PO,PR,SIGI,TH,EP,B,AA(N15),
     *             AA(N16),AA(N17),IX,MTAG,IBC,JBC,NUMR,NUMR1,NNP)
      DO 700 N=1,NPMAX
      NN=2*N
      BO(NN-1)=BO(NN-1)+B(NN-1)
  700 BO(NN)=BO(NN)+B(NN)
C
C          PRINT OUT CUMULATIVE DISPLACEMENTS
C
```

```
SOLV151        WRITE(6,2002)(N,BO(2*N-1),BO(2*N),N=1,NUMNP)
SOLV152        IF(KLK.EQ.1) GO TO 750
SOLV153        IF((KKK.EQ.0).OR.(SR2.GT.1.)) GO TO 800
SOLV154    750 CONTINUE
SOLV155    800 IF(NUMER.GT.0) GO TO 850
SOLV156        IF(NCASE.GT.0) GO TO 850
SOLV157 C
SOLV158 C      IF INITIAL STRESSES ARE EVALUATED IN THIS STEP INITIALIZE BO
SOLV159 C
SOLV160        DO 810 N=1,NEQ
SOLV161    810 BO(N)=0.
SOLV162    850 NCASE=NCASE+1
SOLV163        DO 900 N=1,NELMAX
SOLV164    900 IF(FR(N).GT.1.) FR(N)=1.
SOLV165        RETURN
SOLV166   1000 FORMAT (1H1,40X,'LOAD MATRIX'/(10E13.4))
SOLV167   2000 FORMAT (26HONEGATIVE AREA ELEMENT NO. I4)
SOLV168   2002 FORMAT(12H1 N.P NUMBER 18X 2HUX 18X 2HUY /(I12,2F20.7))
SOLV169        END
```

```
      SUBROUTINE ONED (R,Z,UR,UZ,CODE,BO,F,RO,PR,SIGI,TH,FR,IX,MTAG,
     *                  IBC,JBC,NUMR,NUMR1,NNP)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TOTAL/ AA(30000), IA(7000)
      COMMON/ONE/NUMNP,NUMEL,NUMMAT,NUMPC,NPMAX,NELMAX,N,NNN,KKK,KLK,
     .   ISHIFT,MBAND,NBAND,NUMBLK,MTYPE,NCASE,MCASE,NMR,NL,NPP,NEQ,NPC
     .   ,NP,NSTEP,NCODE
      COMMON/TWO/RR(5),ZZ(5),S(10,10),P(10),ST(3,10),C(4,4),SIG(7),EE(4)
     .   ,TITLE(18),SR1,SR2,XC,YC,ACELR,ACELZ,BCELR,BCELZ,VOL
      COMMON/THREE/STOP,MTOT,NTOT,N13,N14,M8
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),CODE(NUMNP),
     .    BO(NEQ),E(5,NUMMAT),RO(NUMMAT),PR(NPC   ,2),SIGI(NUMEL,8),
     .    TH(NUMEL),FR(NUMEL),IX(NUMEL,5),MTAG(NUMEL),IBC(NPC),
     .    JBC(NPC   ),NUMR(NMR),NUMR1(NMR),NNP(NMR)
C
      DO 100 I=1,8
      P(I)=0.0
      DO 100 J=1,8
  100 S(I,J)=0.0
      MTYPE=IX(N,5)
      I=IX(N,1)
      J=IX(N,2)
      DX=R(J)-R(I)
      DY=Z(J)-Z(I)
      XL=DSQRT(DX**2+DY**2)
      COSA=DX/XL
      SINA=DY/XL
      COMM= E(1,MTYPE)*E(5,MTYPE)/XL
      S(1,1)=COSA*COSA*COMM
      S(1,2)=COSA*SINA*COMM
      S(1,3)=-S(1,1)
      S(1,4)=-S(1,2)
      S(2,1)=S(1,2)
      S(2,2)=SINA*SINA*COMM
      S(2,3)=-S(1,2)
      S(2,4)=-S(2,2)
      S(3,1)=S(1,3)
      S(3,2)=S(2,3)
      S(3,3)=S(1,1)
      S(3,4)=S(1,2)
      S(4,1)=S(1,4)
      S(4,2)=S(2,4)
      S(4,3)=S(3,4)
      S(4,4)=S(2,2)
      II=4
      IF(NNN.EQ.1) II=0
      EP=SIGI(N,II+1)/E(1,MTYPE)
      DX=DX*EP
      DY=DY*EP
      P(1)=S(1,1)*DX+S(1,2)*DY
```

```
ONED 51      P(2)=S(2,1)*DX+S(2,2)*DY
ONED 52      P(3)=-P(1)
ONED 53      P(4)=-P(2)
ONED 54      RETURN
ONED 55      END
```

```
      SUBROUTINE QUAD (R,Z,UR,UZ,CODE,BD,E,RO,PR,SIGI,TH,FR,IX,MTAG,
     *                 IBC,JBC,NUMR,NUMR1,NNP)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TOTAL/ AA(30000), IA(7000)
      COMMON/ONE/NUMNP,NUMEL,NUMMAT,NUMPC,NPMAX,NELMAX,N,NNN,KKK,KLK,
     .    ISHIFT,MBAND,NBAND,NUMBLK,MTYPE,NCASE,MCASE,NMR,NL,NPP,NEQ,NPC
     .    ,NP,NSTEP,NCODE
      COMMON/TWO/RR(5),ZZ(5),S(10,10),P(10),ST(3,10),C(4,4),SIG(7),EE(4)
     .    ,TITLE(18),SR1,SR2,XC,YC,ACELR,ACELZ,BCELR,BCELZ,VOL
      COMMON/THREE/STOP,MTOT,NTOT,N13,N14,M8
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),CODE(NUMNP),
     .    BD(NEQ),E(5,NUMMAT),RO(NUMMAT),PR(NPC   ,2),SIGI(NUMEL,8),
     .    TH(NUMEL),FR(NUMEL),IX(NUMEL,5),MTAG(NUMEL),IBC(NPC),
     .    JBC(NPC   ),NUMR(NMR),NUMR1(NMR),NNP(NMR)
      DIMENSION LM(4),U(3),V(3)
C
      CALL STPSTR (R,Z,UR,UZ,CODE,BD,E,RO,PR,SIGI,TH,FR,IX,MTAG,
     *             IBC,JBC,NUMR,NUMR1,NNP)
      DO 130 J=1,10
      P(J)=0.
      DO 120 I=1,3
  120 ST(I,J)=0.
      DO 130 I=1,10
  130 S(I,J)=0.
      DO 140 I=1,4
      NPN=IX(N,I)
      RR(I)=R(NPN)
  140 ZZ(I)=Z(NPN)
      XC=(RR(1)+RR(2)+RR(3)+RR(4))/4.
      YC=(ZZ(1)+ZZ(2)+ZZ(3)+ZZ(4))/4.
      RR(5)=XC
      ZZ(5)=YC
      K=5
      J=1
      I=4
      LM(3)=9
      NT=4
      IF(IX(N,3).NE.IX(N,4)) GO TO 160
      NT=1
      LM(3)=5
      I=1
      K=3
      J=2
      XC=(RR(1)+RR(2)+RR(3))/3.
      YC=(ZZ(1)+ZZ(2)+ZZ(3))/3.
      RR(5)=RR(3)
      ZZ(5)=ZZ(3)
  160 DO 200 NN=1,NT
      LM(1)=2*I-1
      LM(2)=2*J-1
      U(1)=ZZ(JI-ZZ(K)
      U(2I=ZZ(K)-ZZ(I)
      U(3)=ZZ(I)-ZZ(J)
      V(1)=RR(K)-RR(J)
      V(2)=RR(I)-RR(K)
      V(3)=RR(J)-RR(I)
      AREA=(RR(J)*U(2)+RR(I)*U(1)+RR(5)*U(3))/2.
      VOL=VOL+AREA*TH(N)
      COMM=.25/AREA
      XNT=NT
      COM=2.0/XNT
      COM=COM*COMM
      DX=AREA*TH(N)*RO(MTYPE)/3.
      DY=DX*ACELZ
      DX=DX*ACELR
      DO 180 L=1,3
      II=LM(L)
      ST(1,II)=ST(1,II)+U(L)*COM
      ST(2,II+1)=ST(2,II+1)+V(L)*COM
      ST(3,II)=ST(3,II)+V(L)*COM
      ST(3,II+1)=ST(3,II+1)+U(L)*COM
      P(II)=P(II)+DX
      P(II+1)=P(II+1)+DY
      DO 180 M=1,3
      JJ=LM(M)
      S(II,JJ)=S(II,JJ)+(U(L)*C(1,1)*U(M)+V(L)*C(3,3)*V(MI+V(L)*C(1,3)*U
     .(M)+U(L)*C(1,3)*V(MI)*COMM
      S(II,JJ+1)=S(II,JJ+1)+(U(L)*C(1,2)*V(M)+V(L)*C(3,3)*U(M)+V(L)*C(2,
     .3)*V(M)+U(L)*C(1,3)*U(M))*COMM
      S(II+1,JJ+1I=S(II+1,JJ+1)+(V(L)*C(2,2)*V(M)+U(L)*C(3,3)*U(M)+U(L)*
     .C(2,3)*V(M)+V(L)*C(2,3)*U(M))*COMM
      S(JJ+1,II)=S(II,JJ+1)
  180 CONTINUE
      I=J
      J=J+1
  200 CONTINUE
      IF(IX(N,3).EQ.IX(N,4)) GO TO 250
      DO 240 I=1,2
      KK=ID-I
      DO 240 K=1,KK
      CC=S(KK+1,K)/S(KK+1,KK+1)
      P(K)=P(K)-CC*P(KK+1)
      DO 230 J=1,3
  230 ST(J,KI=ST(J,K)-CC*ST(J,KK+1)
      DO 240 J=1,KK
  240 S(J,K)=S(J,K)-CC*S(J,KK+1)
  250 CONTINUE
      II=0
      IF (NNN.GT.1) II=4
      SIG(1)=-SIGI(N,II+1I
QUAD101          SIG(2)=-SIG1(N,II+2)
QUAD102          SIG(3)=-SIG1(N,II+3)
QUAD103          DO 520 I=1,8
QUAD104          DO 510 J=1,3
QUAD105      510 P(I)=P(I)+ST(J,I)*SIG(J)*VOL
QUAD106      520 CONTINUE
QUAD107          RETURN
QUAD108          END
```

```
      SUBROUTINE STRSTR (R,Z,UR,UZ,CODE,BO,E,RO,PR,SIGI,TH,FR,IX,MTAG,
     *                   IBC,JBC,NUMR,NUMR1,NNP)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TOTAL/ AA(30000), IA(7000)
      COMMON/ONE/NUMNP,NUMEL,NUMMAT,NUMPC,NPMAX,NELMAX,N,NNN,KKK,KLK,
     .    ISHIFT,MBAND,NBAND,NUMBLK,MTYPE,NCASE,MCASE,NMR,NL,NPP,NEQ,NPC
     .    ,NP,NSTEP,NCODE
      COMMON/TWO/RR(5),ZZ(5),S(10,10),P(10),ST(3,10),C(4,4),SIG(7),EE(4)
     .    ,TITLE(18),SR1,SR2,XC,YC,ACELR,ACELZ,BCELR,BCELZ,VOL
      COMMON/THREE/STOP,MTOT,NTOT,N13,N14,MB
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),CODE(NUMNP),
     .    BO(NEQ),E(5,NUMMAT),RO(NUMMAT),PR(NPC   ,2),SIGI(NUMEL,8),
     .    TH(NUMEL),FR(NUMEL),IX(NUMEL,5),MTAG(NUMEL),IBC(NPC),
     .    JBC(NPC   ),NUMR(NMR),NUMR1(NMR),NNP(NMR)
C
      MTYPE=IX(N,5)
      VOL=0.
      DO 50 KK=1,4
   50 EE(KK)=E(KK,MTYPE)
      IF((NCASE.GT.1).AND.(NNN.EQ.1)) GO TO 60
      IF(MTAG(N).GT.0) GO TO 70
   60 CC=EE(2)/(1.-EE(2))
      BB=EE(1)/(1.-EE(2)**2)
      COMM=BB/(1.-CC**2)
      C(1,1)=COMM
      C(1,2)=COMM*CC
      C(1,3)=0.
      C(1,4)=0.
      C(2,1)=C(1,2)
      C(2,2)=C(1,1)
      C(2,3)=0.
      C(2,4)=0.
      C(3,1)=0.
      C(3,2)=0.
      C(3,3)=.5*COMM*(1.-CC)
      C(3,4)= 0.
      C(4,1)= C(1,2)
      C(4,2)= C(1,2)
      C(4,3)= 0.
      C(4,4)= 0.
      CC= 2.*DSIN(EE(4)/57.296)
      BB= 1.732*(3.-DSIN(EE(4)/57.296))
      PP=6.*DCOS(EE(4)/57.296)
      EE(4)=CC/BB
      EE(3)= EE(3)*PP/BB
      GO TO 500
   70 CC= 2.*DSIN(EE(4)/57.296)
      BB= 1.732*(3.-DSIN(EE(4)/57.296))
      PP=6.*DCOS(EE(4)/57.296)
      EE(4)=CC/BB
      FF(3)= FF(3)*PP/BB
      CC=2.*(1.+FF(2))/(3.-6.*FF(2))
      DD=SIGI(N,I)-SIGI(N,2)
      FF=SIGI(N,1)-SIGI(N,4)
      GG=SIGI(N,2)-SIGI(N,4)
      BJ2= (DD**2+FF**2+GG**2)/6. +SIGI(N,3)**2
      BJ2=DSQRT(BJ2)
      BJ1=SIGI(N,1)+SIGI(N,2)+SIGI(N,4)
      DD=BJ1/BJ2
      BB=1.+9.*(FF(4)**2)*CC
      CC=3.*FF(4)*CC-DD/3.
      DD=FF(4)-DD/6.
      H1=.5*CC/(BB*BJ2)
      H2=DD*CC/BB-FF(2)*FF(3)/(BB*BJ2*(1.-2.*FF(2)))
      H3=.5/(BB*BJ2*BJ2)
      BB=FF(1)/(1.+FF(2))
      C(1,1)=BB*(1.-H2-2.*H1*SIGI(N,1)-H3*(SIGI(N,1)**2))
      C(1,2)=-BB*(H2+H1*(SIGI(N,1)+SIGI(N,2))+H3*SIGI(N,1)*SIGI(N,2))
      C(1,3)=-BB*(H1*SIGI(N,3)+H3*SIGI(N,1)*SIGI(N,3))
      C(1,4)=0.
      C(2,1)=C(1,2)
      C(2,2)=BB*(1.-H2-2.*H1*SIGI(N,2)-H3*(SIGI(N,2)**2))
      C(2,3)=-BB*(H1*SIGI(N,3)+H3*SIGI(N,2)*SIGI(N,3))
      C(2,4)=0.
      C(3,1)=C(1,3)
      C(3,2)=C(2,3)
      C(3,3)=BB*(.5-H3*(SIGI(N,3)**2))
      C(3,4)=0.
      C(4,1)=-BB*(H2+H1*SIGI(N,1)+H1*SIGI(N,4)+H3*SIGI(N,1)*SIGI(N,4))
      C(4,2)=-BB*(H2+H1*SIGI(N,2)+H1*SIGI(N,4)+H3*SIGI(N,2)*SIGI(N,4))
      C(4,3)=-BB*(H1*SIGI(N,3)+H3*SIGI(N,3)*SIGI(N,4))
      C(4,4)=0.
  500 RETURN
      END
      SUBROUTINE MODIFY(A,B,NL ,MBAND,NBAND,N,U)
      IMPLICIT REAL*8(A-H ,O-Z)
      DIMENSION B(NL),A(NL,NBAND)
C
      DO 250 M=2,MBAND
      K=N-M+1
      IF(K.LE.0) GO TO 235
      B(K)=B(K)-A(K,M)*U
      A(K,M)=0.0
  235 K=N+M-1
      IF(NL .LT.K) GO TO 250
      B(K)=B(K)-A(N,M)*U
      A(N,M)=0.0
  250 CONTINUE
      A(N,1)=1.0
      B(N)=U
      RETURN
      END
      SUBROUTINE BANSOL (ND,NBAND,MM,NUMBLK,B,A)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION B(ND),A(ND,NBAND)
C
      NN=ND/2
      NL=NN+1
      NH=NN+NN
      REWIND 1
      REWIND 2
      NB=0
      GO TO 150
  100 NB=NB+1
      DO 125 N=1,NN
      NM=NN+N
      B(N)=B(NM)
      B(NM)=0.0
      DO 125 M=1,MM
      A(N,M)=A(NM,M)
  125 A(NM,M)=0.0
      IF(NUMBLK.EQ.NB) GO TO 200
  150 READ (2) (B(N),(A(N,M),M=1,MM),N=NL,NH)
      IF(NB.EQ.0) GO TO 100
  200 DO 300 N=1,NN
      IF(A(N,1).EQ.0.) GO TO 300
      B(N)=B(N)/A(N,1)
      DO 275 L=2,MM
      IF (A(N,L).EQ.0.) GO TO 275
      C=A(N,L)/A(N,1)
      I=N+L-1
      J=0
      DO 250 K=L,MM
      J=J+1
  250 A(I,J)=A(I,J)-C*A(N,K)
      B(I)=B(I)-A(N,L)*B(N)
      A(N,L)=C
  275 CONTINUE
  300 CONTINUE
      IF(NUMBLK.EQ.NB) GO TO 400
      WRITE (1) (B(N),(A(N,M),M=2,MM),N=1,NN)
      GO TO 100
  400 DO 450 M=1,NN
      N=NN+1-M
      DO 425 K=2,MM
      L=N+K-1
  425 B(N)=B(N)-A(N,K)*B(L)
      NM=N+NN
      B(NM)=B(N)
  450 A(NM,NB)=B(N)
      NB=NB-1
      IF(NB.EQ.0) GO TO 500
```

```
BANS 51         BACKSPACE 1
BANS 52         READ (1) (B(N),(A(N,M),M=2,MM),N=1,NN)
BANS 53         BACKSPACE 1
BANS 54         GO TO 400
BANS 55     500 K=0
BANS 56         DO 600 NB=1,NUMBLK
BANS 57         DO 600 N=1,NN
BANS 58         K=K+1
BANS 59         NM=N+NN
BANS 60     600 B(K)=A(NM,NB)
BANS 61         RETURN
BANS 62         END
```

```
      SUBROUTINF STRFSS (R,Z,UR,UZ,CODF,BO,F,RO,PR,SIGI,TH,FR,B,EP,
     *                      RATIO,NEW,IX,MTAG,IBC,JBC,NUMR,NUMR1,NNP)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TOTAL/ AA(30000), IA(7000)
      COMMON/ONE/NUMNP,NUMEL,NUMMAT,NUMPC,NPMAX,NELMAX,N,NNN,KKK,KLK,
     .    ISHIFT,MBAND,NBAND,NUMBLK,MTYPE,NCASE,MCASE,NMR,NL,NPP,NEQ,NPC
     .    ,NP,NSTEP,NCODE
      COMMON/TWO/RR(5),ZZ(5),S(10,10),P(10),ST(3,10),C(4,4),SIG(7),EE(4)
     .    ,TITLE(18),SR1,SR2,XC,YC,ACELR,ACELZ,BCELR,BCELZ,VOL
      COMMON/THREE/STOP,MTOT,NTOT,N13,N14,M8
      DIMENSION R(NUMNP),Z(NUMNP),UR(NUMNP),UZ(NUMNP),CODE(NUMNP),
     .    BO(NEQ),E(5,NUMMAT),RO(NUMMAT),PR(NPC   ,2),SIGI(NUMEL,8),
     .    TH(NUMEL),FR(NUMEL),IX(NUMEL,5),MTAG(NUMEL),IBC(NPC),
     .    JBC(NPC   ),NUMR(NMR),NUMR1(NMR),NNP(NMR)
      OIMENSION EP(NUMEL,4),RATIO(NUMEL),NEW(100),TT(4),TF(4)
      DIMENSION B(NEQ)
C
      TOLL=0.
      SR=1.
      SRATIO=1.0
      NMY=0
      MPRINT=0
      KK=0
      NOPT=0
      TOLLA=10.
      KJK=0
      KLK=0
      IF((NCASE.LE.1).OR.(NNN.GT.1)) GO TO 60
      OO 50 N=1,NELMAX
      IF (MTAG(N).GT.0) KJK=KJK+1
   50 CONTINUE
   60 CONTINUE
      DO 300 N=1,NELMAX
      RATIO(N)=1.
      IX(N,5)=IABS(IX(N,5))
      MTYPE=IX(N,5)
      IF((RO(MTYPE).EQ.0.).OR.(IX(N,3).EQ.IX(N,2))) GO TO 300
      CALL QUAO (R,Z,UR,UZ,COOE,BO,E,RO,PR,SIGI,TH,FR,IX,MTAG,
     *              IBC,JBC,NUMR,NUMR1,NNP)
      MM=4
      IF(IX(N,3).EQ.IX(N,4)) MM=3
      DO 175 I=1,4
      EP(N,I) = 0.
      DO 175 J=1,MM
      II=2*J
      JJ=2*IX(N,J)
  175 EP(N,I)=EP(N,I)+ST(I,II)*B(JJ) + ST(I,II-1)*B(JJ-1)
      OO 190 I=1,4
      SIG(I)=0.0
      OO 190 J=1,3
  190 SIG(I(=SIG(I(+C(I,J)*FP(N,J)
      DO 195 I=1,4
      II=I+4
  195 SIGI(N,II(=SIGI(N,I(+SIG(I(
      DD=SIGI(N,5(-SIGI(N,6(
      FF=SIGI(N,6)-SIGI(N,8)
      GG=SIGI(N,5(-SIGI(N,8)
      AJ2= (DD**2+GG**2+FF**2(/6.0 +SIGI(N,7)**2
      AJ2=DSQRT(AJ2(
      AJ1=SIGI(N,5(+S(GI(N,6)+SIGI(N,8)
      FAIL=AJ2+FF(4(*AJ1
      IF(MTAG(N(.EQ.0) GO TO 200
      IF (FF(3(.EQ.0.( GO TO 300
      IF ((NCASE.GT.1).AND.(NNN.EQ.1(( GO TO 198
      DD=DABS(FAIL-FF(3((
      CHECK =0.05*FF(3(
      IF(DD.LE.CHECK) GO TO 300
      KKK=1
      CR=CHECK/DD
      IF(CR.GE.SR) GO TO 300
      SR=CR
      NOPT=1
      JJJ=N
      GO TO 300
  198 IF(FAIL.GT.FF(3I( GO TO 300
      MTAG(N(=0
      KJK=KJK-1
      GO TO 300
  200 CONTINUE
      IF(FAIL.LT.FF(3)( GO TO 300
      KK=KK+1
      DD=SIGI(N,1)-SIGI(N,2)
      FF=SIGI(N,1)-SIGI(N,4(
      GG=SIGI(N,2(-SIGI(N,4)
      AJ2= (DD**2+GG**2+FF**2)/6.0 +SIGI(N,3)**2
      AJ1=SIGI(N,1(+SIGI(N,2)+SIGI(N,4)
      DD=SIGI(N,5)-SIGI(N,6)
      FF=SIGI(N,6)-SIGI(N,8(
      GG=SIGI(N,5(-SIGI(N,8)
      BJ2= (DD**2+GG**2+FF**2)/6.0 +SIGI(N,7)**2
      BJ1=SIGI(N,5)+SIGI(N,6)+SIGI(N,8)
      CCC=(SIGI(N,I(-SIGI(N,2))*(SIGI(N,5)-SIGI(N,6))+(SIGI(N,2(-SIGI(N,
     .4))*(SIGI(N,6(-SIGI(N,8()+(SIGI(N,4)-SIGI(N,1()*(SIGI(N,8)-SIGI(N,
     .5((
      CCC=CCC/6,0+ SIGI(N,3)*SIGI(N,7(
      AB=AJ2-(FF(4(*AJ1(**2
      BB=BJ2-(FF(4(*BJ1(**2
      CC=CCC-(FF(4(*FF(4I*AJ1*BJI(
      DD=FF(4(*FF(3(*AJ1
      FF=FF(4(*FF(3(*BJI
STRF101          GG=FF(3)*EE(3)
STRF102          AAA=AB+BB-2.*CC
STRF103          BBB=AB-CC+DD-FF
STRF104          CCC=2.*DD-GG+AB
STRF105          GGG=BBB*BBB-AAA*CCC
STRF106          IF(GGG.LT.0.)   WRITE(6,2008) N
STRF107          IF(GGG.LT.0.)  GGG=DABS(GGG)
STRF108          GGG=DSQRT(GGG)
STRF109          IF(AAA.NE.0.) GO TO 220
STRF110          RATIO(N)=.5*CCC/BBB
STRF111          GO TO 300
STRF112      220 AB=BBB/AAA
STRF113          BB=DABS(GGG/AAA)
STRF114          RATIO(N)=AB-BB
STRF115          IF(RATIO(N).LT.0.) RATIO(N)=AB+BB
STRF116          IF(RATIO(N).GE.1.) RATIO(N)=.99999
STRF117          IF(RATIO(N).LT.0.) RATIO(N)=0.
STRF118          IF (KK.NE.1) GO TO 240
STRF119          WRITE (6,3000)
STRF120      240 WRITE (6,3005) N,RATIO(N)
STRF121      300 CONTINUE
STRF122          IF(KK.EQ.0) SR=1.0
STRF123          IF(KK.EQ.0) GO TO 410
STRF124          DO 350 N=1,NELMAX
STRF125          IF(MTAG(N).GT.0) GO TO 350
STRF126          MTYPE=IX(N,5)
STRF127          IF (RO(MTYPE).EQ.0.) GO TO 350
STRF128          DD=RATIO(N)
STRF129          IF(DD.GE.SR) GO TO 350
STRF130          SR=DD
STRF131          NOPT=2
STRF132          NMY=N
STRF133          KKK=1
STRF134      350 CONTINUE
STRF135          WRITE (6,3010) SR,SR2
STRF136          IF(NNN.EQ.1) GO TO 352
STRF137          DD= SR*(1.0-SR2)
STRF138          IF(DD.LT.0.03)  NOPT=3
STRF139          IF(DD.LT.0.03) DD=0.03
STRF140          SR= DD/(1.0-SR2)
STRF141          DD= SR*(1.0-SR2)
STRF142          IF(DD.GT.0.10)  NOPT=4
STRF143          IF(DD.GT.0.10)  DD=0.10
STRF144          SR=DD/(1.-SR2)
STRF145          IF(SR.GT.1.0) SR=1.00
STRF146      352 CONTINUE
STRF147          SRATIO=SR
STRF148          IF (KJK.GT.0) SR=0.
STRF149      410 CONTINUE
STRF150          DO 420 N=1,NPMAX
```

```
      II=2*N-1
      B(II)=B(II)*SR
  420 B(II+1)=B(II+1)*SR
      WRITE (6,4010) NMY,SR,SR2
      NOO=0
      FRMAX=0.
      DO 600 N=1,NELMAX
      MTYPE=IX(N,5)
      IF (RO(MTYPE).EQ.0.) GO TO 515
      IF(IX(N,3).NE.IX(N,2)) GO TO 430
C
C        CALCULATE STRESSES IN ONE DIMENSIONAL ELEMENTS
C
      I=IX(N,1)
      J=IX(N,2)
      XC=(R(I)+R(J))/2.0
      YC=(Z(I)+Z(J))/2.0
      DX=R(J)-R(I)
      DY=Z(J)-Z(I)
      XL=DSQRT(DX**2+DY**2)
      DU=B(2*J-1)-B(2*I-1)
      DV=B(2*J)-B(2*I)
      DL=DV*DY/XL+DU*DX/XL
      DO 422 I=1,7
  422 SIG(I)=0.
      DX=E(1,MTYPE)*DL/XL
      SIG(1)=DX*SR
      IF(E(3,MTYPE).EQ.1.) GO TO 425
      SIGI(N,5)=-DX*(1-SR)
      SIGI(N,1)=SIG(1)+SIGI(N,1)
      GO TO 515
  425 SIG(5)=-DX*(1.-SR)
      SIG(1)=SIGI(N,1)+DX*SR
      DY=E(4,MTYPE)/(SIGI(N,1)+DX)-1.
      IF(DABS(DY).LE..05) GO TO 428
      II=1
      IF(NNN.GT.1) II=5
      DZ=SIGI(N,1)+DX
      DX=SIGI(N,II)+DX
      DY=(E(4,MTYPE)-DZ)*SIGI(N,II)/DX
      SIGI(N,1)=SIG(1)+DY
      SIGI(N,5)=SIG(5)+DY
      KLK=1
      SIG(5)=SIGI(N,5)
      SIG(4)=SIGI(N,1)
      GO TO 515
  428 SIGI(N,1)=SIG(1)
      SIGI(N,5)=SIG(5)
      GO TO 515
  430 I=IX(N,1)
```

```
      J=IX(N,2)
      K=IX(N,3)
      L=IX(N,4)
      MTYPF=IX(N,5)
      IF(K.FQ.L) GO TO 440
      XC=(R(I)+R(J)^R(K)+R(L))/4.
      YC=(Z(I)+Z(J)+Z(K)+Z(L))/4.
      GO TO 445
  440 XC=(R(I)+R(J)+R(K))/3.
      YC=(Z(I)+Z(J)+Z(K))/3.
  445 CONTINUF
      XN=0.5
      IF(NNN.EQ.1) XN=1.0
      IF(MTAG(N).LE.0)  XN=1.0
      OO 450 I=1,4
      II=I+4
      TT(I) = SIGI(N,I)
      TF(I)=SIGI(N,II)
      SIG(I)=SIGI(N,II)-SIGI(N,I)
      SIGI(N,II)=SIG(I)*(1.-SR)
      SIGI(N,II)=-SIGI(N,II)
      SIG(I)= XN*SIG(I)*SR+SIGI(N,I)
  450 SIGI(N,I)=SIG(I)
      IF((NMY.EQ.0).OR.(KJK.GT.0))  GO TO 485
      IF (MTAG(N).LE.0) GO TO 481
      ISTOP=0
      DO 470 JJ=1,5
      CALL STRSTR (R,Z,UR,UZ,CODE,BO,E,RO,PR,SIGI,TH,FR,IX,MTAG,
     *             IBC,JBC,NUMR,NUMR1,NNP)
      DO 455 I=1,4
      II=I+4
      SIGI(N,II)=TT(I)
      OO 455 J=1,3
  455 SIGI(N,II)=SIGI(N,II)+C(I,J)*EP(N,J)*SR
      DO=(SIGI(N,5)-SIGI(N,6))**2+(SIGI(N,6)- SIGI(N,8))**2+(SIGI(N,5)-
     .SIGI(N,8))**2
      AJ2=DD/6.0+ SIGI(N,7)**2
      AJ2=DSQRT(AJ2)
      AJ1=SIGI(N,5)+SIGI(N,6)+SIGI(N,8)
      FAIL=AJ2+FE(4)*AJ1
      IF(JJ.EQ.1) GO TO 460
       D=DABS(FAIL-FAIL1)
      TOL=0.005*FAIL
      IF(DO.LF.TOL)  ISTOP=1
  460 FAIL1 =FAIL
      OO 465 I=1,4
      II=I+4
      SIG(I)=SIGI(N,II)-TT(I)
  465 SIGI(N,I)=TT(I)+SIG(I)*XN
      IF(N.NE.1) GO TO 451
      WRITE(6,1000) (SIGI(N,I),I=1,8)
 1000 FORMAT(10H SIGI(N,I) / (8F12.4))
  451 CONTINUE
      IF(ISTOP.GT.0) GO TO 475
  470 CONTINUE
  475 DO 480 I=1,4
      II=I+4
      SIGI(N,I)=SIGI(N,II)
      SIGI(N,II)=-(TF(I)-SIGI(N,I))
      SIG(I)=SIGI(N,I)
  480 CONTINUE
  481 TR=SRATIO
      TOL=0.05*TR
      IF(TR.GE.0.95) GO TO 485
      IF(TOL.LT.0.005) TOL=0.005
      IF(MTAG(N).GT.0) GO TO 485
      IF(RATIO(N).LE.TR) GO TO 482
      DD=RATIO(N)-TR
      IF(DD.GT.TOL) GO TO 485
  482 NOD=NOD+1
      NEW(NOD)=N
      MTAG(N)=1
  485 CONTINUE
      DO 490 I=1,4
  490 FF(I)=F(I,MTYPE)
      CC= 2.*DSIN(FF(4)/57.296)
      BB= 1.732*(3.-DSIN(FF(4)/57.296))
      PP=6.*DCOS(FF(4)/57.296)
      FF(4)=CC/BB
      FF(3)= FF(3)*PP/BB
      SIG(7)=SIGI(N,4)
      CC=(SIG(1)+SIG(2))/2.
      BB=(SIG(1)-SIG(2))/2.
      CR=DSQRT(BB**2+SIG(3)**2)
      SIG(4)=CC+CR
      SIG(5)=CC-CR
      SIG(6)=0.0
      IF ((BB.EQ.0.0).AND.(SIG(3).EQ.0.0)) GO TO 510
      SIG(6)=28.648*DATAN2(SIG(3),BB)
  510 CONTINUE
      DD=(SIG(1)-SIG(2))**2+(SIG(2)-SIG(7))**2+(SIG(7)-SIG(1))**2
      AJ2=DD/6. +SIG(3)**2
      AJ2=DSQRT(AJ2)
      AJ1=SIG(1)+SIG(2)+SIG(7)
      FAIL=FF(3)-FF(4)*AJ1
      FR(N)=AJ2/FAIL
      IF (MTAG(N).GT.0) GO TO 515
      IF (KJK.GT.0) GO TO 515
      IF (FR(N).GE.0.99) MTAG(N)=1
      IF(MTAG(N).EQ.0) GO TO 515
```

```
STRE301          NOO=NOO+1
STRE302          NEW(NOO)=N
STRE303      515 IF(MPRINT.GT.0) GO TO 550
STRE304          WRITE(6,2000) NNN
STRE305          MPRINT=50
STRE306      550 MPRINT=MPRINT-1
STRE307          IF((IX(N,2)-IX(N,3)).EQ.0)  FR(N)=0.0
STRE308          IF(TH(N).NE.1.) SIG(7)=0.0
STRE309          IF (RO(MTYPE).NE.0.) GO TO 555
STRE310          WRITE (6,2010) N
STRE311          GO TO 600
STRE312 C
STRE313 C            PRINT OUT STRESSES
STRE314 C
STRE315      555 IF(MTAG(N).LT.1) GO TO 556
STRE316          WRITE(6,2011) N,XC,YC,(SIG(I),I=1,7),MTAG(N),FR(N),MTYPE
STRE317          GO TO 557
STRE318      556 WRITE(6,2001) N,XC,YC,(SIG(I),I=1,7),MTAG(N),FR(N),MTYPE
STRE319      557 IF(IX(N,2).EQ.IX(N,3)) GO TO 600
STRE320          IF((FR(N).GE.1.).OR.(FR(N).LT.FRMAX)) GO TO 570
STRE321          FRMAX=FR(N)
STRE322          NMAX=N
STRE323      570 CONTINUE
STRE324          DD=SIGI(N,5)-SIGI(N,6)
STRE325          FF=SIGI(N,6)-SIGI(N,8)
STRE326          GG=SIGI(N,5)-SIGI(N,8)
STRE327          BJ2= (DD**2+GG**2+FF**2)/6.0 +SIGI(N,7)**2
STRE328          BJ2=DSQRT(BJ2)
STRE329          TOLL=TOLL+BJ2
STRE330      600 CONTINUE
STRE331          SR2= (SR1*SR) + SR2
STRE332          SR1= (1.0-SR) * SR1
STRE333          WRITE(6,2002)TOLL,SR,NMY,KK,SR2
STRE334          WRITE(6,5000) FRMAX,NMAX
STRE335          IF(NOO.EQ.0) GO TO 620
STRE336          WRITE(6,2020) (NEW(I),I=1,NOO)
STRE337      620 KKK=1
STRE338          IF(TOLL.LE.TOLLA) KKK=0
STRE339          NKT=NELMAX/2
STRE340          JCK=0
STRE341          DO 705 N=1,NELMAX
STRE342          IF(MTAG(N).GT.0) JCK=JCK+1
STRE343      705 CONTINUE
STRE344          IF(JCK.LT.NKT) GO TO 710
STRE345          WRITE(6,2007)
STRE346          CALL EXIT
STRE347      710 CONTINUE
STRE348          IF(NNN.EQ.1)  GO TO 800
STRE349          GO TO (750,760,770,780),NOPT
STRE350          GO TO 800
```

```
  750 WRITF(6,2003) JJJ
      GO TO 800
  760 WRITF(6,2004)
      GO TO 800
  770 WRITF(6,2005)
      GO TO 800
  780 WRITF(6,2006)
  800 RFTURN
 2000 FORMAT(1H1/
     .36H STRFSSES AFTER ARPROX(MATION NUMBFR I4////
     .7H FL.NO. 7X 1HX 7X 1HY 4X 8HX-STPESS 4X 8HY-STRFSS 3X 9HXY-STRESS
     . 2X 10HMAX-STRFSS 2X 10HMIN-STRFSS 7H  ANGLF 4X 8HZ-STRFSS 3X 7HPL
     .ASTIC 3X 4HFAIL 3X 5HMTYPF     )
 2001 FORMAT (I7,2F8.2,1P5E12.4,0P1F7.2,1PF12.4, I6,0P1F11.3 ,I6 )
 2002 FORMAT(39HOTHF UNBALANCFD LOAD AT THIS STAGF IS  F14.5//
     .47H THF RATIO FOR CORRECTION OF STORED STRESSFS IS F10.4//
     .31H THF NFXT FLEMENT YIFLDING IS     I4/
     .91H AND THE TOTAL NUMBER OF FLEMENTS THAT CAN YIFLD WITH THF LINFA
     .R ADDITION OF TOTAL LOAD IS       I4/
     .50H LOAD UP TO THIS STAGF AS A FRACTION OF TOTAL IS      F20.5   )
 2003 FORMAT(I10HOSTRFSS RATIO GOVFRNFD BY STRFSS STATF BFING MORF THAN
     .5 PFRCFNT OUTSIDE YIELD SURFACE FOR ELEMENT NUMBER =          I5)
 2004 FORMAT( 53HO STRFSS RATIO GOVFRNED BY NFXT FLFMENT Y(FLDING      )
 2005 FORMAT(10CHOSTRESS RAT(O GOVERNED BY THE MINIMUM VALUE  OF SR FOR
     . ANY STFP                                                       )
 2006 FORMAT(110HO STRFSS RATIO GOVFRNED BY THF MAXIMUM VALUF OF STRFSS
     . RATIO FOR ANY STFP                                             )
 2007 FORMAT(64HO JOB TFRMINATFD AS HALF OF TOTAL ELEMFNTS YIFLD AT THIS
     . STFP          )
 2008 FORMAT( 110H ARGUMFNT NEGATIVF IN THE EQUATION FOR CALCULATING
     . THF VALUF OF STRESS RATIO FOR FLFMENT NUMBFR  =                I5)
 2009 FORMAT(45HO STRFSS RATIO SR FOR THIS CYCLE=                 F10.5)
 2010 FORMAT(I7,50H THIS ELEMFNT HAS BFFN REMOVFD FROM THF ANALYSIS  )
 2011 FORMAT (I7,2F8.2,1P5F12.4,0P1F7.2,1PF12.4, I6,0P1F11.3 ,I6,3X,1H*)
 2020 FORMAT(48HO THF FOLLOWING NEW FLFMENTS YIFLD IN TH(S STFP      //
     .20(5)
 3000 FORMAT (35HO THF FOLLOWING FLFMENTS CAN YF(LD                  //
     .34HO FLEMENT NO.           RATIO(N)                             )
 3005 FORMAT ((10,F20.5)
 3010 FORMAT( 17HO SR          SR2                                    /
     . 2F10.5)
 4010 FORMAT(6HO NMY=  I5,3HSR=  F20.5, 4HSR2=  E20.5     )
 5000 FORMAT(I7HOMAXIMUM FAIL (S  F6.3,17H FOR FLEMENT NO.   (5)
      FND
```

CHAPTER III: GRIFTH - A Computer Program for Two-Dimensional Analysis of Progressive Failure of Rock Following Griffith and Modified Griffith Theory

3.1. Purpose and Capability

This computer program is applicable to plane stress or plane strain analysis of stresses, deformations and progressive fracture in elastic brittle rock following Griffith and modified Griffith theory. Arbitrary initial stresses, arbitrary sequence of construction or excavation, arbitrary history of load application can be simulated. One dimensional elements are included. The program is applicable to study of fracture initiation and propagation in arbitrary elastic brittle structure systems composed of several different materials. Non-monotonic loading is considered.

Theoretical development incorporated in the program is documented in Part I-Technical Report of this report.

3.2. Program Organization

The computer program is in Fortran language. Files 1 and 2 are used to store system equations and element properties respectively. Tapes 5 and 6 are the input/output files. The program capacity can be altered by changing the dimensions of arrays AA and IA. These correspond to the total locations required for real and integer arrays respectively. NTOT, MTOT at lines MAIN 29 , MAIN 30 are set equal to the dimensions of AA and IA.

The program consists of the following units:

a. MAIN

In this unit, the control information including maximum number of elements, nodal points, different materials, boundary pressure cards, the number of steps of excavation or construction, the maximum number of elements removed or added to the system at any stage is read in. This information is used to organize the dimensions of various arrays. This done, further processing of data is done in subroutine INPT.

b. Subroutine INPT

This subroutine is called by the unit MAIN. The first step is to read in material property data for all different materials in the system. Nodal point coordinates, loads and code descriptors for boundary conditions are read in or generated. Element geometry, initial stresses, initial crack openings, if any, are read in or generated. Maximum band width for the system is calculated and dimensions of blocks for generation and storage of system stiffness defined. After defining these controls, the incremental structure is analyzed in steps. For each step the number of nodal points, the number of elements, the number of elements and nodal points removed or added, if any, the number of boundary pressure cards and the material type of the elements added or changes in material properties are read in. After the information is assembled the solution process is transferred to subroutine SOLVE.

c. Subroutine SOLVE

This subroutine called by INPT is concerned with obtaining stresses, deformations and sequence of progressive fracture of elements in a given step of loading/construction/excavation. To trace progressive fracture, the solution process traces a sequence of elements reaching fracture along with the effects of stress redistribution associated with fracture leading to secondary fractures. The process consists of applying the total load and then scaling it according to the minimum ratio of load increment needed to ensure one element reaching fracture. Once an element fractures, the associated stress redistribution will result in secondary fractures at the same total load. This is referred to as system stability iteration in the program.

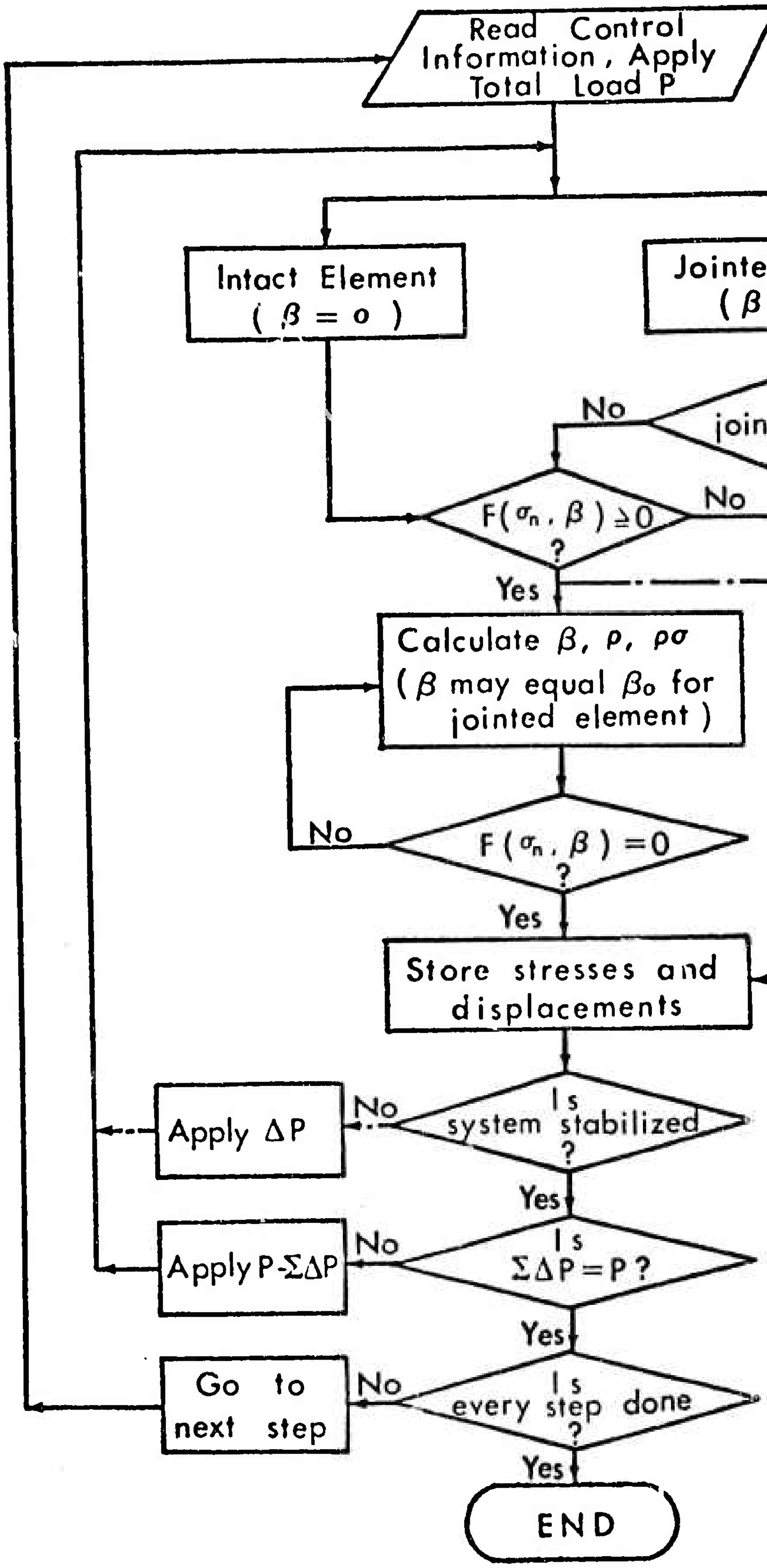
The SOLVE subroutine calls ONED and QUAD to obtain element stiffness for one or two-dimensional elements respectively. This information is stored on File 2 and is updated in case of fracture or failure. Solution to the stiffness equations is obtained in subroutine BANSOL. Subroutine STRESS defines the stresses corresponding to a load application. This is referred to as the initial state for any load increment. As several elements may crack during a load increment, it is necessary to scale it to pin-point the sequential fracture phenomenon. This is accomplished in subroutine SCALE.

d. Subroutine SCALE

This subroutine calls subroutine GRIFTH to check each element for fracture or closure of cracks. The stress ratio for each element is calculated, if necessary, using interpolation (subroutine INTER). Iterations to define a value of orientation of fracture are referred to as beta-stability iterations. After choosing the minimum stress ratio applicable and the element that next fractures, system stability iteration is accomplished to define all secondary fractures associated with the primary fracture. This process is illustrated in the flow chart as Figure III-1.

3.3. Input Data

a. Job Title (18A4). This card will give the descriptive identification for the job.



Read Control Information, Apply Total Load P
Intact Element (β = o)
Jointed Element (β = β₀)
Is joint open ?
No
Yes
F(σn, β) ≥ 0 ?
No
Yes
Calculate β, ρ, ρσ (β may equal β₀ for jointed element)
Calculate β Release σ₀ Modify [k]
No
F(σn, β) = 0 ?
Yes
Store stresses and displacements
Apply ΔP
No
Is system stabilized ?
Yes
Apply P-ΣΔP
No
Is ΣΔP = P ?
Yes
Go to next step
No
Is every step done ?
Yes
END

Figure 3.1. Flow Chart

Analysis of Progressive Fracture of Rock

b. Control Information (4I5, 3F10.2, 3I5)

Information	Columns
Maximum number of nodal points	1-5
Maximum number of elements	6-10
Number of different materials	11-15
Maximum number of pressure cards	16-20
Body force in X-direction	21-30
Body force in Y-direction	31-40
Reference (stress-free) temperature	41-50
Number of initially open cracks	51-55
Code to designate plane stress or plane strain NPLANE = 1 for plane stress = 2 for plane strain	56-60
Total number of excavation & construction steps	61-65

c. Material Property Cards. One set of cards must be provided for each material.

In each set:

i. First card (2I5, F10.3, 2I5) will give the following information:

Information	Columns
Material identification number	1-5
Number of temperature cards (8 maximum)	6-10
Mass density of the material	11-20
Material code to designate materials which will follow the fracture criteria	21-25

code = 1 for materials which will follow the fracture criteria
= 0 for materials which will not fracture

Material code to identify the initial anisotropy due to initial cracks code = 1 for initial anisotropy = 0 for isotropy	26-30

ii. If columns 21-25 in card i is not zero, the following information must be provided (3F10.3)

Information	Columns
Tensile strength	1-10
Internal frictional coefficient	11-20
Tolerance for crack closure	21-30

iii. Subsequent cards, one for each temperature, the number being defined in columns 6-10 of the first card, will carry the following information (F10.0, E10.0, 2F10.0):

Information	Columns
Temperature	1-10
Elastic modulus	11-20
Poisson's ratio	21-30
Coefficient of thermal expansion	31-40

iv. If columns 26-30 in card i is not zero, the following information must be providied (4F10.4):

Information	Columns
Modulus ratio	1-10
Angle of fault	11-20
Frictional coefficient	21-30
Shear strength	31-40

d. Nodal Point Cards (I5, F5.0, 5F10.0). One card for each nodal point with the following information:

Information	Columns
Nodal point number	1-5
Type of nodal point	6-10
X-ordinate	11-20
Y-ordinate	21-30
XR	31-40
XZ	41-50
Temperature	51-60

If the number in columns 6-10 is:

0 = XR is the specified X-load and XZ is the specified Y-load

1 = XR is the specified X-displacement and XZ is the specified Y-load

2 = XR is the specified X-load and XZ is the specified Y-displacement

3 = XR is the specified X-displacement and XZ is the specified Y-displacement.

All loads are considered to be total forces acting on an element of unit thickness. Nodal point cards must be in numerical sequence. If cards are omitted, the omitted nodal points are generated at equal intervals along a straight line between the defined nodal points. The necessary temperatures are determined by linear interpolation. The type of the nodal point, as well as XR, XZ, are set equal to zero.

e. Element Material Cards (16I5). These cards shall carry the material type of all the elements. Each card shall have material types for 16 elements in sequence. The material type for each element must be read in as no interpolation has been provided for.

f. Elements Cards (5I5, 5X, 3F10.0). One card for each element will provide the following data:

Information	Columns
Number of element	1-5
Nodal point I	6-10
Nodal point J	11-15
Nodal point K	16-20
Nodal point L	21-25
Initial stresses:	
Component in X-direction	31-40
Component in Y-direction	41-50
Shearing stress on X-Y planes	51-60

Nodal points I, J, K, L are corners of each individual element in a counter-clockwise order for a right handed system of coordinates. For triangular elements set nodal point L same as nodal point K. The element cards must be in the numerical sequence. Any cards that are omitted will be automatically generated in the program by incrementing each of the I, J, K and L nodal points by one. The material type will be taken same as for the last element defined.

g. Initial Cracks Cards. If columns 41-45 in card b is not zero, the following information must be provided for each initial crack (2I5, F10.0).

Information	Columns
Element number	1-5

Tag number	6-10
Crack Angle	11-20

The tag number is used to designate the crack mode

Tag = 1 for single crack
    = 2 for double crack

The crack angle is defined as the angle between X-axis and the normal to the crack plane, positive counter-clockwise.

h. Incremental Step Information. One set of cards must be provided for each step of construction or dismantling. Construction and dismantling may not be mixed in one step.

i. First card (18A4). This gives the descriptive title for the step for which information follows.

ii. Second card (7I5). Following information is given for the step which is described in Title.

Information	Columns
Number of nodal points in this step	1-5
Number of elements in this step	6-10
Number of elements removed or added with reference to previous step	11-15
Number of pressure boundary cards (total pressure for this step)	16-20
New material type for the elements added or eliminated	21-25
Code to designate addition to structure of dismantling	26-30

Code = 0 for dismantling
     = 1 for construction

Number of nodal points included in analysis but not part of structure at this step	31-35

iii. Addition or Dismantling of Structure. One or more cards shall indicate the numbers of elements removed or added in the step under consideration (16I5).

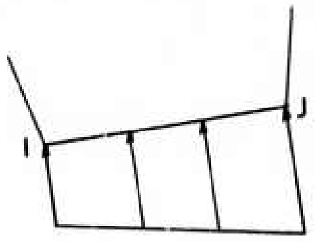
The total number of elements included here must be the same as indicated in columns 11-15 of card h-ii.

iv. Nodal Points Included in Analysis but not Taking Any Load (16I5). One or more cards shall indicate the numbers of the nodal points included in the analysis at this step but not forming part of the load carrying system.

The total number of nodal points listed here must be the same as indicated in columns 31-35 of card h-ii.

v. Pressure Boundary Cards (2I5, 2F10.0). If there are any boundary pressures for this step, then one card for each boundary element which is subjected to normal pressure will carry the following information:

Information	Columns
Nodal Point I	1-5
Nodal Point J	6-10
Total Normal Pressure at I	11-20
Total Normal Pressure at J	21-30



As shown in the sketch, the boundary element must be on the left as one progresses from I to J. Surface tensile force is input as a negative pressure.

j. Last Card (A6). The last card at the end of data deck is "stop" card. It carries the characters STOP in columns 1 through 4.

3.4. Output Information

The following information is developed and printed by the program:

a. Print out of problem data. This includes information in material properties, mesh layout, geometry and boundary conditions, loads and constraints, initial cracks, etc.

b. Initial stresses before any excavation or construction at the center of each element.

c. The incremental as well as cumulative nodal point displacements and the stresses after application of a load increment.

d. The stresses upon application of total load increment.

e. The stresses, crack orientation, type of crack, at the first element cracking under a load increment.

f. The stresses, crack orientation, type of crack for each beta-iteration to define the correct crack orientation.

g. The stresses, crack orientation, type of crack for each system stability iteration to define secondary fractures following the first fracture in a load increment .

h. The stress ratio as a proportion of total load at each stability iteration.

3.5. Fortran Listing

```
C     ****************************************************************
C     * PRUGRAM IDENTIFICATION: PROGRAM PFA                          *
C     * PROGRAMMER: S.W. HUANG,I.S.RAI, THE OHIO STATE UNIVERSITY    *
C     * PURPOSE: PROGRESSIVE FRACTURE ANALYSIS                       *
C     * FAILURE CRITERIA: FRACTURE ACCORDING TO GRIFFITH AND MODIFIED *
C     *            GRIFFITH THEORIES                                 *
C     *  THE FORMULATION IS DOCUMENTED IN THE FINAL REPORT DATED MARCH *
C     * 31,1973, ON CONTRACT H0210017 BETWEEN THE OHIO STATE UNIVERSITY*
C     * AND THE UNITED STATES BUREAU OF MINES SUPPORTED BY THE ADVANCED*
C     * RESEARCH PROJECTS AGENCY. INSTRUCTIONS FOR USE OF THE PROGRAM *
C     * ARE CONTAINED IN PART II OF THE REPORT.                      *
C     ****************************************************************
C
      COMMON AA(16000),IA(4500)
      COMMON/ONE/ NUMNP,NUMEL,NUMMAT,NUMPC,NPC,MBAND,NUMBLK,NL,MTYPE,N,
     .   VOL,ACELR,ACELZ,Q,HED(18),STOP,SR,SRI,TOTAL,TOL,TOL1,XC,YC,
     .   TEMP,SIGN,SIGDI,SIGDJ,
     .   LLL,III,JJJ,JCK,KCHECK,JCHECK,NPLANE,NUMER,NCODE,JA,NEQ
     .   ,NBAND,NCRACK,NSTEP,N15,M7,NTOT
      COMMON/TWO/ C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(51,2215),
     .   LM(4),EE(3),EPS(3)
      COMMON/THREE/  E(8,4,8),EO(4,8),TENS(8),XNU(8),RO(8),EPST(8),
     .   MTC(8),NIC(8),MID(8)
      DEFINE FILE 1(100,1500,U,NBK),2(510,230,U,ID)
      CALL ERRSET(208,256,-1,1)
C
C     READ AND WRITE CONTROL INFORMATION FOR THE PROBLEM
C
      NTOT=16000
      MTOT=4500
      TOL1=0.001
      READ (5,1000) HED,NUMNP,NUMEL,NUMMAT,NUMPC,ACELR,ACELZ,Q,NCRACK,
     .     NPLANE,NSTEP
      IF(NPLANE.EQ.1) WRITE(6,2000)
      IF(NPLANE.EQ.2) WRITE(6,2005)
      NEQ=2*NUMNP
      WRITE (6,2010) HED,NUMNP,NUMEL,NUMMAT,NUMPC,ACELR,ACELZ,Q,NSTEP
      NPC=NUMPC
      IF(NPC.EQ.0) NPC=1
      N1=1
      N2=N1+NUMNP
      N3=N2+NUMNP
      N4=N3+NUMNP
      N5=N4+NEQ
      N6=N5+NEQ
      N7=N6+NEQ
      N8=N7+NEQ
      N9=N8+NUMNP
      N10=N9 +6*NUMEL
      N11=N10+6*NUMEL
      N12=N11+  NUMEL
      N13=N12+  NUMEL
      N14=N13+  NUMEL
      N15=N14+2*NPC
      M1=1
      M2=M1+5*NUMEL
      M3=M2+NUMEL
      M4=M3+NUMEL
      M5=M4+NUMEL
      M6=M5+NPC
      M7=M6+NPC
      JJ=M7-MTOT
      IF(JJ.LE.0) GO TO 100
      WRITE (6,3000)  JJ
      CALL EXIT
  100 CONTINUE
      CALL INPT  (AA(N1),AA(N2),AA(N3),AA(N4),AA(N5),AA(N6),AA(N7),
     .   AA(N8),AA(N9),AA(N10),AA(N11),AA(N12),AA(N13),AA(N14),IA(M1),
     .   IA(M2),IA(M3),IA(M4),IA(M5),IA(M6))
      STOP
 1000 FORMAT (18A4/4I5,3F10.2,3I5)
 2000 FORMAT(1H1,' PLANE STRESS ANALYSIS')
 2005 FORMAT(1H1,' PLANE STRAIN ANALYSIS')
 2010 FORMAT (1H0 18A4/
     1 30H0 NUMBER OF NODAL POINTS------ I3 /
     2 30H0 NUMBER OF ELEMENTS---------- I3 /
     3 30H0 NUMBER OF DIFF. MATERIALS--- I3 /
     4 30H0 NUMBER OF PRESSURE CARDS--   I3/
     5 30H0 X-ACCELERATION-------------- E12.4/
     6 30H0 Y-ACCELERATION-------------- E12.4/
     7 30H0 REFERENCE TEMPERATURE------- E12.4/
     8 30H0 NUMBER OF STEPS------------- I3 )
 3000 FORMAT (70H PROGRAM EXECUTION TERMINATED. REQUIRED CORE EXCEEDS MT
     .OT BY                                                         I10)
      END
      SUBROUTINE INPT   (R,Z,CODE,UU,CU,BI,BJ,T,SIGI,EPSI,SIGNM,BETA,
     .   RATIO,PR,IX,MTAG,NTAG,JNT,IBC,JBC)
C
      COMMON AA(16000),IA(4500)
      COMMON/ONE/ NUMNP,NUMEL,NUMMAT,NUMPC,NPC,MBAND,NUMBLK,NL,MTYPE,N,
     .   VOL,ACELR,ACELZ,Q,HED(18),STOP,SR,SRI,TOTAL,TOL,ICLI,XC,YC,
     .   TEMP,SIGN,SIGDI,SIGDJ,
     .   LLL,III,JJJ,JCK,KCHECK,JCHECK,NPLANE,NUMER,NCODE,JA,NEQ
     .   ,NBAND,NCRACK,NSTEP,N15,M7,NTOT
      COMMON/TWO/ C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),
     .   LM(4),EE(3),EPS(3)
      COMMON/THREE/  E(8,4,8),EO(4,8),TENS(8),XNU(8),RO(8),EPST(8),
     .   MTC(8),NTC(8),MID(8)
      DIMENSION TITLE(18),NNP(200),MAD(200),NUMR(200)
      DIMENSION R(NUMNP),Z(NUMNP),CODE(NUMNP),UU(NUMNP,2),CU(NEQ),
     .BI(NEQ    (NEQ),T(NUMNP),SIGI(NUMEL,6),EPSI(NUMEL,6),SIGNM(NUMEL),
     .BETA(NU    ),RATIO(NUMEL),PR(NPC,2),IX(NUMEL,5),MTAG(NUMEL),
     .NTAG(NUM   ),JNT(NUMEL),IBC(NPC),JBC(NPC)
      DO 50 M=1,NUMMAT
      READ (5,1010) MTYPE,NTC(MTYPE),RO(MTYPE),MTC(MTYPE),MID(MTYPE)
      WRITE(6,2010) MTYPE,NTC(MTYPE),RO(MTYPE),MTC(MTYPE),MID(MTYPE)
      IF(MTC(MTYPE).EQ.0) GO TO 45
      READ(5,1015)   TENS(MTYPE),XNU(MTYPE),EPST(MTYPE)
      WRITE(6,2015)  TENS(MTYPE),XNU(MTYPE),EPST(MTYPE)
   45 CONTINUE
      NUMTC=NTC(MTYPE)
      READ (5,1020) ((E(I,J,MTYPE),J=1,4),I=1,NUMTC)
      WRITE (6,2020) ((E(I,J,MTYPE),J=1,4),I=1,NUMTC)
      IF(MID(MTYPE).EQ.0) GO TO 50
      READ (5,1022) (EO(K,MTYPE),K=1,4)
      WRITE(6,2022) (EO(K,MTYPE),K=1,4)
      EO(2,MTYPE)=EO(2,MTYPE)/57.296
   50 CONTINUE
C
C     READ NODAL POINT DATA,GENERATE INTERMEDIATE POINTS AND WRITE
C
      WRITE (6,2025)
      L=0
   60 READ  (5,1025) N,CODE(N),R(N),Z(N),UU(N,1),UU(N,2),T(N)
      IF(N.EQ.1) GO TO 70
      ZX=N-L
      DR=(R(N)-R(L))/ZX
      DZ=(Z(N)-Z(L))/ZX
      DT=(T(N)-T(L))/ZX
   70 L=L+1
      IF(N-L) 100,90,80
   80 CODE(L)=0.0
      R(L)=R(L-1)+DR
      Z(L)=Z(L-1)+DZ
      T(L)=T(L-1)+DT
      UU(L,1)=0.
      UU(L,2)=0.
      GO TO 70
   90 IF(NUMNP-N) 100,110,60
  100 WRITE (6,2035) N
      CALL EXIT
  110 WRITE(6,2030) (N,CODE(N),R(N),Z(N),UU(N,1),UU(N,2),T(N),N=1,NUMNP)
C
C     READ ELEMENT DATA , GENERATE INTERMEDIATE ELEMENTS AND WRITE
C
      READ(5,1030) (IX(N,5),N=1,NUMEL)
      WRITE (6,2040)
      N=0
  130 READ(5,1035) M,(IX(M,I),I=1,4)
  140 N=N+1
      IF (M-N) 170,170,150
  150 IX(N,1)=IX(N-1,1)+1
      IX(N,2)=IX(N-1,2)+1
      IX(N,3)=IX(N-1,3)+1
      IX(N,4)=IX(N-1,4)+1
  170 CONTINUE
      SIGI(N,1)=0.
      SIGI(N,2)=0.
      SIGI(N,3)=0.
      MTAG(N)=0
      JNT(N)=0
      BETA(N)=0.
      IF (M-N) 180,180,140
  180 IF (NUMEL-N) 190,190,130
  190 WRITE (6,2045) (N,(IX(N,I),I=1,5),(SIGI(N,I),I=1,3),N=1,NUMEL)
      IF(NCRACK.EQ.0) GO TO 194
      WRITE(6,2050)
      DO 192 M=1,NCRACK
      READ (5,1040) N,MTAG(N),BETA(N)
      WRITE(6,1040) N,MTAG(N),BETA(N)
  192 BETA(N)=BETA(N)/57.296
  194 CONTINUE
C
C     INITIALIZE STRAINS
C
      DO 196 N=1,NUMEL
      NTAG(N)=0
      DO 196 I=1,3
  196 EPSI(N,I)=0.0
C
C     INITIALIZE CUMULATIVE DISPLACEMENTS
C
      DO 198 I=1,NEQ
  198 CU(I)=0.0
      JJ=0
```

```
INPT101       DO 199 I=1,NUMEL
INPT102       DO 199 J=1,4
INPT103       DO 199 K=1,4
INPT104       KK=IABS(IX(I,J)-IX(I,K))
INPT105       IF(KK.GT.JJ) JJ=KK
INPT106   199 CONTINUE
INPT107       NBAND=2*JJ+2
INPT108       NL=(NTOT-N15+1)/(NBAND+1)
INPT109       NLL=NEQ+3
INPT110       IF(NL.GT.NLL) NL=NLL
INPT111       NL=NL/4
INPT112       ND=2*NL
INPT113       NL=4*NL
INPT114       IF(ND.GE.NBAND) GO TO 209
INPT115       WRITE (6,4010) NL,NBAND
INPT116       CALL EXIT
INPT117   209 N16=N15+NL
INPT118       N17=N16+NL*NBAND
INPT119       JJ=N17-NTOT
INPT120       IF(JJ.LE.0) GO TO 210
INPT121       WRITE (6,3000) JJ
INPT122       CALL EXIT
INPT123   210 WRITE (6,4000) N17,M7,NL,NBAND
INPT124 C
INPT125 C     READ CONTROL INFORMATION FOR THE NEXT STEP IN INCREMENTAL ANALYSIS
INPT126 C
INPT127 C     NUMBER OF NODAL POINTS MISSING  NPMIS IS INTRODUCED TO TAKE
INPT128 C     CARE OF EMPTY NODAL POINTS IN THE PREVIOUS STEP AND NEW NODAL
INPT129 C     POINTS INTRODUCED AS A CONSEQUENCE OF CONSTRUCTION OR EXCAVATION
INPT130 C     DONE DURING THIS STEP. NPMIS IS TAKEN AS THE SUM OF EMPTY NODAL
INPT131 C     POINTS DURING PREVIOUS STEP PLUS THE NODAL POINTS VACATED OR
INPT132 C     ADDED INTHIS STEP UPTO  NPMAX  .
INPT133 C
INPT134       NCASE=0
INPT135   200 READ(5,1045) TITLE
INPT136       WRITE(6,2055) TITLE
INPT137       READ(5,1050) NPMAX, NELMAX, NUMER, NUMPC, MTYPE, NCODE ,NPMIS,MADD
INPT138       LLL=1
INPT139       III=0
INPT140       JJJ=0
INPT141       TOTAL=0.0
INPT142       SIGDI=0.
INPT143       SIGDJ=0.
INPT144   202 IF(NUMER.EQ.0) GO TO 204
INPT145       READ(5,1030)  (NUMR(N), N= 1,NUMER)
INPT146   204 IF(NPMIS.EQ.0) GO TO 206
INPT147       READ(5,1030)  (NNP(M), M = 1,NPMIS)
INPT148 C
INPT149 C     DETERMINE BANDWITH
INPT150 C
```

```
  206 NUMEL=NELMAX
      NUMNP=NPMAX
      J=0
      DO 208 N=1,NUMEL
      DO 208 I=1,4
      DO 208 L=1,4
      KK=IABS(IX(N,I)-IX(N,L))
      IF (KK-J) 208,208,207
  207 J=KK
  208 CONTINUE
      MBAND=2*J+2
      IF (NPMIS.EQ.0) GO TO 215
      DO 212 I = 1, NPMIS
      J = NNP(I)
      CU(2*J-1) = 0.
  212 CU(2*J) = 0.
C
C     PRINT OUT CONTROL INFORMATION FOR THE CURRENT STEP IN INCREMENTAL
C     ANALYSIS
C
  215 IF(NCODE.EQ.1) GO TO 240
      IF(NUMER.EQ.0) GO TO 220
      WRITE(6,2060)NCASE,NPMAX,NELMAX,NUMPC,NPMIS,(NUMR(N),N=1,NUMER )
      GO TO 230
  220 WRITE (6,2060) NCASE,NPMAX,NELMAX,NUMPC,NPMIS
  230 IF(NPMIS.EQ.0)  GO TO 270
      WRITE(6,2065) (NNP(M) ,M = 1,NPMIS)
      GO TO 270
  240 IF(NUMER.EQ.0) GO TO 250
      WRITE(6,2070)NCASE,NPMAX,NELMAX,NUMPC,NPMIS,(NUMR(N),N=1,NUMER   )
      GO TO 260
  250 WRITE(6,2070)NCASE,NPMAX,NELMAX,NUMPC,NPMIS
  260 IF(NPMIS.EQ.0)  GO TO 270
      WRITE(6,2065) (NNP(M) ,M = 1,NPMIS)
C
C     IF THERE ARE ANY BOUNDARY PRESSURES FOR THIS STEP READ AND PRINT
C     THIS DATA
C
  270 IF (NUMPC) 290,310,290
  290 WRITE (6,2075)
      DO 300 L=1,NUMPC
      READ(5,1060) IBC(L),JBC(L),PR(L,1),PR(L,2)
  300 WRITE(6,2080) IBC(L),JBC(L),PR(L,1),PR(L,2)
  310 CONTINUE
C
C     CHANGE MATERIAL TYPE FOR ORIGINAL ELEMENTS
C
      IF(MADD.EQ.0) GO TO 336
      WRITE(6,2082)
      READ(5,1030) (MAD(I),IX(MAD(I),5),I=1,MADD)
```

```
INPT201       WRITE(6,2083)(MAD(I),IX(MAD(I),5),I=1,MADD)
INPT202       DO 335 I=1,MADD
INPT203       N=MAD(I)
INPT204       DO 335 J=1,3
INPT205       SIGIIN,J)=0.
INPT206   335 EPSIIN,J)=0.
INPT207   336 CONIINUE
INPT208 C
INPT209 C     IF INITIAL STRESS CONDIIION IS TO BE ANALYSED INSTEAD OF INPUT,
INPT210 C     USF NUMER = 0 AND DIRECTLY PROCEED TO STIFF.
INPT211 C
INPT212 C
INPT213 C     CORRECT MATERIAL TYPE FOR ELEMENTS REMOVED OR ADDED. SET INITIAL
INPT214 C     STRESSES EQUAL TO ZERO
INPT215 C
INPT216       IF INUMER.FQ.0) GO TO 340
INPT217       DO 330 I=1,NUMER
INPT218       NUM = NUMR(I)
INPT219       IXINUM,5)=MTYPE
INPT220       DO 330 J=1,3
INPT221       SIGI(NUM,J) = 0.0
INPT222   330 EPSIINUM,J)=0.0
INPT223   340 SR=0.
INPT224   345 CONTINUE
INPT225       DO 350 N=1,NUMEL
INPT226   350 RATIOIN)=1.0
INPT227 C
INPT228 C     FORM STIFFNESS MATRIX IN BLOCKS FOR THIS STEP
INPT229 C
INPT230       CALL        SOLVF (R,Z,CODE,UU,CU,BI,BJ,T,SIGI,EPSI,SIGNM,BETA,
INPT231      .   RATIO,PR,AA(N15),AA(N16),IX,MTAG,NTAG,JNT,IBC,JBC)
INPT232       IFISTOP.FQ.1.) CALL EXIT
INPT233 C
INPT234 C     SOLVE MATRIX EQUATIONS
INPT235 C
INPT236       CALL BANSOL (AA(N15),AAIN16),MBAND,NBAND,NUMBLK,NL,JA)
INPT237       CALL        STRESS(R,Z,CODE,UU,CU,BI,BJ,T,SIGI,EPSI,SIGNM,BETA,
INPT238      .   RATIO,PR,AA(N15),AAIN16),IX,MTAG,NTAG,JNT,IBC,JBC)
INPT239 C
INPT240 C
INPT241       CALL SCALF (IX,MTAG,NTAG, JNT,RATIO,BETA,SIGI,SIGNM,EPSI,CU,
INPT242      .   AA(N15),AAIN16))
INPT243 C
INPT244 C
INPT245       JJJ=1
INPT246       IF(JCHECK.EQ.0) GO TO 400
INPT247       III=III+1
INPT248       WRITE(6,2084) LLL,III
INPT249       NKT=0
INPT250       NML=NUMEL/2
```

```
      DO 380 N=1,NUMEL
      IF(MTAG(N).GT.2) NKT=NKT+1
      IF(NKT.LT.NML) GO TO 380
      WRITE(6,2085) NKT,NML
      CALL EXIT
  380 CONTINUE
      GO TO 345
  400 IF(TOTAL.EQ.1.0  ) GO TO 600
      LLL=LLL+1
      III=0
  450 GO TO 345
  600 CONTINUE
      IF(NCASE.GE.1) GO TO 800
C
C     IF THE INITIAL STRESSES ARE EVALUATED IN THIS STEP
C    1INITIALIZE  CU
C
      DO 750 I = 1,NEQ
  750 CU(I) = 0.0
  800 NCASE = NCASE + 1
      IF(NCASE.LE.NSTEP) GO TO 200
 1010 FORMAT (2I5,1F10.3,2I5)
      RETURN
 1015 FORMAT(3F10.3)
 1020 FORMAT(F10.3,E10.0,2F10.4)
 1022 FORMAT(4F10.4)
 1025 FORMAT (I5,F5.1,5F10.4)
 1030 FORMAT(16I5)
 1035 FORMAT(5I5)
 1040 FORMAT(2I5,F10.5)
 1045 FORMAT(18A4)
 1050 FORMAT(8I5)
 1055 FORMAT (16I5)
 1060 FORMAT(2I5,2F10.3)
 2010 FORMAT (17H0MATERIAL NUMBER= I3, 30H, NUMBER OF TEMPERATURE CARDS=
     1 I3, 15H, MASS DENSITY= E12.4,16H, MATERIAL CODE= I3
     2,16H, MATERIAL I.D.= I3)
 2015 FORMAT(18H0TENSILE STRENGTH=E12.4,21H, COEFF. OF FRICTION=E12.4
     *,19H, INITIAL OPENING = E12.4)
 2020 FORMAT (15H0  TEMPERATURE 10X 5HE     9X 6HNU     5X  8HALPHA    /
     *(F15.2,3E15.3))
 2022 FORMAT(15H0 MODULUS RATIO,15H ANGLE OF FAULT,15H FRICTION COEF.,
     *15H SHEAR STRENGTH /(4E15.3))
 2025 FORMAT (108H1NODAL POINT         TYPE  X ORDINATE  Y ORDINATE  X LO
     1AD OR DISPLACEMENT  Y LOAD OR DISPLACEMENT TEMPERATURE )
 2030 FORMAT (I12,F12.2,2F12.5,2E24.7,F12.3)
 2035 FORMAT (26H0NODAL POINT CARD ERROR N= I5)
 2040 FORMAT(120H1ELEMENT NO.      I     J     K     L    MATERIAL
     1SIGIXX      SIGIYY       SIGIXY                                 )
 2045 FORMAT (I13,4I6,I12,3F12.3)
```

```
INPT301  2050 FORMAT(1H0,' INITIALLY CRACKED ELEMENTS'/,'  EL. MTAG      BETA'/)
INPT302  2055 FORMAT(1H1 18A4/////)
INPT303  2060 FORMAT( ' RESULTS AFTER STEP NO.-------------------------------',I5/
INPT304       1' TOTAL NUMBER OF NODAL POINTS IN SYSTEM AT THIS STEP--------',I5/
INPT305       2' TOTAL NUMBER OF   ELEMENTS   IN SYSTEM AT THIS STEP--------',I5/
INPT306       3' TOTAL NUMBER OF PRESSURE CARDS AT THIS STEP ---------------',I5/
INPT307       4' TOTAL NUMBER OF NODAL POINTS MISSING AT THIS STEP----------',I5/
INPT308       5' ELEMENTS REMOVED IN THIS STEP ARE ' /(20I5))
INPT309  2065 FORMAT(     '  NUMBERS OF NODAL POINTS MISSING IN THIS STEP ARE   '/
INPT310       1(20I5))
INPT311  2070 FORMAT( ' RESULTS AFTER STEP NO.-------------------------------',I5/
INPT312       1' TOTAL NUMBER OF NODAL POINTS IN SYSTEM AT THIS STEP--------',I5/
INPT313       2' TOTAL NUMBER OF   ELEMENTS   IN SYSTEM AT THIS STEP--------',I5/
INPT314       3' TOTAL NUMBER OF PRESSURE CARDS AT THIS STEP ---------------',I5/
INPT315       4' TOTAL NUMBER OF NODAL POINTS MISSING AT THIS STEF----------',I5/
INPT316       5' ELEMENTS ADDED   IN THIS STEP ARE ' /(20I5))
INPT317  2075 FORMAT(29H0PRESSURE BOUNDARY CONDITIONS/40H     I     J    PRESSUR
INPT318       1E I    PRESSURE J)
INPT319  2080 FORMAT(2I6,2F14.3)
INPT320  2082 FORMAT(///' MATERIAL TYPES CHANGED AT THIS STEP'//'  EL.  MTYPE')
INPT321  2083 FORMAT(2I5/)
INPT322  2084 FORMAT(1H1,' START ITERATION TO OBTAIN SYSTEM STABILITY UNDER THE
INPT323       *LOAD INCREMENT NO.',I3/
INPT324       *' SYSTEM STABILITY ITERATION NO.',I3//)
INPT325  2085 FORMAT(' NUMBER OF ELEMENTS CRACKED -',I4,'- REACHES THE LIMIT-',I
INPT326       *4,'- PROGRESSIVE FRACTURE IS LEADING THE SYSTEM TO TOTAL FAILURE')
INPT327  3000 FORMAT (70H PROGRAM EXECUTION TERMINATED. REQUIRED CORE EXCEEDS MT
INPT328       .OT BY                                                  I10)
INPT329  4000 FORMAT (47H FOR THIS PROGRAM THE LOCATION USED IN AA IS =     I5,
INPT330       .    17H AND IN IA IS =          I5/
INPT331       .    18H MAX BLOCK SIZE =          I5/
INPT332       .    18H MAX BAND WIDTH =          I5/ )
INPT333  4010 FORMAT (25H0 NL IS LESS THAN 2*MBAND      /
INPT334       .    5H0 NL=             I5/
INPT335       .    8H0 MBAND=          I5)
INPT336        END
```

```
      SUBROUTINE SOLVE (R,Z,CODE,UU,CU,BI,BJ,T,SIGI,EPSI,SIGNM,BETA,
     .   RATIO,PR,B,A,IX,MTAG,NTAG,JNT,IBC,JBC)
C
      COMMON/ONE/ NUMNP,NUMEL,NUMMAT,NUMPC,NPC,MBAND,NUMBLK,NL,MTYPE,N,
     .   VOL,ACELR,ACELZ,Q,HED(18),STOP,SR,SR1,TOTAL,TOL,TOLI,XC,YC,
     .   TEMP,SIGN,SIGDI,SIGDJ,
     .   LLL,III,JJJ,JCK,KCHECK,JCHECK,NPLANE,NUMER,NCODE,JA,NEQ
     .   ,NBAND,NCRACK,NSTEP,N15,M7,NTOT
      COMMON/TWO/ C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),
     .   LM(4),EE(3),EPS(3)
      COMMON/THREE/  E(6,4,8),EO(4,8),TENS(8),XNU(8),RO(8),EPST(8),
     .   MTC(8),NTC(8),MID(8)
      DIMENSION R(NUMNP),Z(NUMNP),CODE(NUMNP),UU(NUMNP,2),CU(NEQ),
     .BI(NEQ),BJ(NEQ),T(NUMNP),SIGI(NUMEL,6),EPSI(NUMEL,6),SIGNM(NUMEL),
     .BETA(NUMEL),RATIO(NUMEL),PR(NPC,2),IX(NUMEL,5),MTAG(NUMEL),
     .NTAG(NUMEL),JNT(NUMEL),IBC(NPC),JBC(NPC)
      DIMENSION B(NL),A(NL,NBAND)
      NB=NL/4
      ND=2*NB
      ND2=2*ND
      STOP=0.0
      NUMBLK=0
      NBK=1
      JA=ND *(MBAND+1)/1500+1
      DO 50 N=1,ND2
      B(N)=0.0
      DO  50 M=1,MBAND
   50 A(N,M)=0.0
   60 NUMBLK=NUMBLK+1
      NH=NB*(NUMBLK+1)
      NM=NH-NB
      NLL=NM-NB+1
      KSHIFT=2*NLL-2
      DO 210 N=1,NUMEL
      MTAGI=MTAG(N)
      ICHECK=0
      LCHECK=1
      ID=N
      MM=4
      IF(IX(N,3).EQ.IX(N,4)) MM=3
      IF(IX(N,3).EQ.IX(N,2)) MM=2
      IF (IX(N,5)) 210,210,65
   65 DO 80 I=1,4
      IF (IX(N,I)-NLL) 80,70,70
   70 IF (IX(N,I)-NM) 90,90,80
   80 CONTINUE
      GO TO 210
   90 IF(NTAG(N).EQ.1) GO TO 92
      IF(MTAG(N).GT.2) GO TO 99
      IF((LLL.GT.1).AND.(MTAG(N).EQ.0)) GO TO 99
      IF((III).GT.0).AND.(MTAG(N).EQ.0)) GO TO 99
   92 IF(IX(N,3)-IX(N,2)) 95,85,95
   85 CALL        ONED  (R,Z,CODE,UU,CU,BI,BJ,T,SIGI,EPSI,SIGNM,BETA,
     .   RATIO,PR,B,A,IX,MTAG,NTAG,JNT,IBC,JBC)
      GO TO 98
   95 CALL        ELEMEN(R,Z,CODE,UU,CU,BI,BJ,T,SIGI,EPSI,SIGNM,BETA,
     .   RATIO,PR,B,A,IX,MTAG,NTAG,JNT,IBC,JBC)
      NTAG(N)=0
   98 ICHECK=1
C     ****************************************************************
C     * WRITE ELEMENT INFORMATION ON FILE 2                          *
C     ****************************************************************
      WRITE(2'10) ((C(IIK,JJK),JJK=1,3),EE(IIK),IIK=1,3)
     1 ,((S(JJI,KKI),KKI=1,8),JJI=1,8),((ST(IKK,JKK),JKK=1,8),IKK=1,3)
     2 ,(RR(JII),ZZ(JII),JII=1,4),XC,YC,TEMP,VOL,MTYPE,N
   99 IX(N,5)=-IX(N,5)
      IF((LLL.GT.1).OR.(III.GT.0)) GO TO 101
      IF(VOL) 100,100,101
  100 WRITE(6,2000) N
      STOP=1.0
C     ****************************************************************
C     *   CALCULATE NODAL POINT FORCES                               *
C     ****************************************************************
  101 IF((LLL.EQ.1).AND.(III.EQ.0)) GO TO 145
      IF(ICHECK.EQ.1) GO TO 105
      READ(2'10)  ((C(IIK,JJK),JJK=1,3),EE(IIK),IIK=1,3)
     1 ,((S(JJI,KKI),KKI=1,8),JJI=1,8),((ST(IKK,JKK),JKK=1,8),IKK=1,3)
     2 ,(RR(JII),ZZ(JII),JII=1,4),XC,YC,TEMP,VOL,MTYPE,N
      GO TO 166
C
C     EVALUATE STRESS TRANSFORMATION MATRIX AND STRESSES TO BE RELEASED
C        FOR NEWLY FRACTURED ELEMENT ONLY
C
  105 IF(IX(N,2).EQ.IX(N,3)) GO TO 166
      DO 106 I=1,3
      IF(SIGI(N,I).NE.0.) GO TO 110
  106 CONTINUE
      ICHECK=0
      GO TO 154
  110 IF (MTAG(N).NE.0) GO TO 111
      SIG(1)=0.
      SIG(2)=0.
      SIG(3)=0.
      GO TO 154
  111 IF(MTAG(N).NE.4) GO TO 113
      DO 112 I=1,3
      SIG(I)=-SIGI(N,I)
  112 SIGI(N,I)=0.
      GO TO 154
  113 EPSX=BETA(N)
```

```
      CC=COS(EPSX)
      SS=SIN(EPSX)
      S2=SS*SS
      C2=CC*CC
      SC=SS*CC
      SC2=2.D*SC
      CSD=C2-S2
      SIGXX=C2*SIGI(N,1)+S2*SIGI(N,2)+SC2*SIGI(N,3)
      SIGYY=S2*SIGI(N,1)+C2*SIGI(N,2)-SC2*SIGI(N,3)
      SIGXY=-SC*(SIGI(N,1)-SIGI(N,2))+CSD*SIGI(N,3)
      DX=EE(2)*SIGXX
      SIG(1)=C2*SIGXX+S2*DX-SC2*SIGXY
      SIG(2)=S2*SIGXX+C2*DX+SC2*SIGXY
      SIG(3)=SC*(SIGXX-DX)+CSD*SIGXY
      DO 144 I=1,3
      SIGI(N,I)=SIGI(N,1)-SIG(I)
  144 SIG(I)=-SIG(1)
      GO TO 154
C
C     CALCULATE TEMPERATURE STRESSES ,FOR LLL=1 AND III=D ONLY
C
  145 DT=TEMP-Q
      DX=EE(3)*DT
      SIG(1)=-(C(1,1)+C(1,2))*DX+SIGI(N,1)
      SIG(2)=-(C(2,2)+C(1,2))*DX+SIGI(N,2)
      SIG(3)=SIGI(N,3)
  154 IF(IX(N,2).EQ.IX(N,3)) GO TO 166
      DO 160 I=1,8
      P(I)=0.0
      IF(LCHECK.EQ.0) GO TO 160
      DO 155 J=1,3
  155 P(1)=P(I)-ST(J,I)*SIG(J)
  160 P(I)=P(I)*VOL
      IF((LLL.GT.1).OR.(III.GT.0)) GO TO 166
C
C     CALCULATE BODY FORCES, FOR LLL=1 AND III=0 ONLY
C
  163 XMM=MM
      DY=VOL*ACELZ*RO(MTYPE)/XMM
      DX=VOL*ACELR*RO(MTYPE)/XMM
      DO 165 I=1,MM
      P(2*I)=P(2*1)+DY
  165 P(2*I-1)=P(2*1-1)+DX
  166 CONTINUE
  167 DO 168 I=1,MM
  168 LM(I)=2*IX(N,I)-2
      DO 200 I=1,MM
      DO 200 K=1,2
      II=LM(I)+K-KSHIFT
      KK=2*I-2+K
      IF(ICHECK.EQ.0) GO TO 172
      B(II)=B(II)+P(KK)
  172 DO 200 J=1,MM
      DO 200 L=1,2
      JJ=LM(J)+L-II+1-KSHIFT
      LL=2*J-2+L
      IF(JJ) 200,200,175
  175 IF(ND-JJ) 180,195,195
  180 WRITE (6,2001) N
      STOP=1.0
      GO TO 210
  195 A(II,JJ)=A(II,JJ)+S(KK,LL)
  200 CONTINUE
  210 CONTINUE
      IF((LLL.GT.1).OR.(III.GT.0)) GO TO 301
      DO 220 N=NLL,NM
      IF(N-NUMNP) 215,215,220
  215 K=2*N-KSHIFT
      B(K)=B(K)+UU(N,2)
      B(K-1)=B(K-1)+UU(N,1)
  220 CONTINUE
      IF(NUMPC) 225,301,225
  225 DO 300 L=1,NUMPC
      I=IBC(L)
      J=JBC(L)
      DR=Z(I)-Z(J)
      DZ=R(J)-R(I)
      PP2=(PR(L,2)+PR(L,1))/6.
      PP1=PP2+PR(L,1)/6.
      PP2=PP2+PR(L,2)/6.
      II=2*I-KSHIFT
      JJ=2*J-KSHIFT
      IF(II) 265,265,235
  235 IF(II-ND) 240,240,265
  240 B(II-1)=B(II-1)+PP1*DR
      B(II)=B(II)+PP1*DZ
  265 IF(JJ) 300,300,270
  270 IF(JJ-ND) 275,275,300
  275 B(JJ-1)=B(JJ-1)+PP2*DR
      B(JJ)=B(JJ)+PP2*DZ
  300 CONTINUE
  301 CONTINUE
      II=ND*(NUMBLK-1)+1
      JJ=ND*NUMBLK
      KK=0
      IF(III.NE.0) GO TO 303
      DO 302 N=II,JJ
  302 BJ(N)=0.
  303 IF((LLL.EQ.1).AND.(III.EQ.0)) GO TO 305
      GO TO 313
  305 DO 306 N=I1,JJ
      KK=KK+1
  306 B1(N)=B(KK)
      GO TO 310
  313 FACTOR=SR
      IF(LL1.EQ.0) FACTOR=1.-TOTAL
      IF(LL1.EQ.1) FACTOR=TOTAL
      DO 314 N=I1,JJ
      KK=KK+1
      BJ(N)=BJ(N)+B(KK)
  314 B(KK)=FACTOR*B1(N)+BJ(N)
  310 DO 400 M=NLL,NH
      IF (M-NUMNP) 315,315,400
  315 U=UU(M,1)
      N=2*M-1-KSHIFT
      IF (CODE(M)) 390,400,316
  316 IF (CODE(M)-1.) 317,370,317
  317 IF (CODE(M)-2.) 318,390,318
  318 IF (CODE(M)-3.) 390,380,390
  370 CALL MODIFY(A,B,ND2,MBAND,NBAND,N,U)
      GO TO 400
  380 CALL MODIFY(A,B,ND2,MBAND,NBAND,N,U)
  390 U=UU(M,2)
      N=N+1
      CALL MODIFY(A,B,ND2,MBAND,NBAND,N,U)
  400 CONTINUE
C     ******************************************************************
C     *  WRITE BLOCK INFORMATION ON FILE 1                             *
C     ******************************************************************
      WRITE(1'NBK)(B(N),(A(N,M),M=1,MBAND),N=1,ND)
      NBK=NBK+JA
      DO 420 N=1,ND
      K=N+ND
      B(N)=B(K)
      B(K)=0.0
      DO 420 M=1,MBAND
      A(N,M)=A(K,M)
  420 A(K,M)=0.0
      IF (NH-NUMNP) 60,480,480
  480 CONTINUE
      RETURN
 2000 FORMAT (26HONEGATIVE AREA ELEMENT NO. I4)
 2001 FORMAT (29HOBAND WIDTH EXCEEDS ALLOWABLE I4)
      END
      SUBROUTINE ONED  (R,Z,CODE,UU,CU,BI,BJ,T,SIGI,EPSI,SIGNM,BETA,
     .   RATIO,PR,B,A,IX,MTAG,NTAG,JNT,IBC,JBC)
      COMMON/ONE/ NUMNP,NUMEL,NUMMAT,NUMPC,NPC,MBAND,NUMBLK,NL,MTYPE,N,
     .   VOL,ACELR,ACELZ,Q,HED(18),STOP,SR,SR1,TOTAL,TOL,TOL1,XC,YC,
     .   TEMP,SIGN,SIGDI,SIGDJ,
     .   LLL,III,JJJ,JCK,KCHECK,JCHECK,NPLANE,NUMER,NCODE,JA,NEQ
     .   ,NBAND,NCRACK,NSTEP,N15,N7,NTOT
      COMMON/TWO/ C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),
     .   LM(4),EE(3),EPS(3)
      COMMON/THREE/  E(8,4,8),LD(4,8),TENS(8),XNU(8),RO(8),EPST(8),
     .   MTC(8),NTC(8),MOD(8)
      DIMENSION R(NUMNP),Z(NUMNP),CODE(NUMNP),UU(NUMNP,2),CU(NEQ),
     .BI(NEQ),BJ(NEQ),T(NUMNP),SIGI(NUMEL,6),EPSI(NUMEL,6),SIGNM(NUMEL),
     .BETA(NUMEL),RATIO(NUMEL),PR(NPC,2),IX(NUMEL,5),MTAG(NUMEL),
     .NTAG(NUMEL),JNT(NUMEL),IBC(NPC),JBC(NPC)
      DIMENSION B(NL),A(NL,NBAND)
C
      DO 100 I=1,8
      P(I)=0.0
      DO 100 J=1,8
  100 S(I,J)=0.0
      MTYPE=IX(N,5)
      I=IX(N,1)
      J=IX(N,2)
      DX=R(J)-R(I)
      DY=Z(J)-Z(I)
      XL= SQRT(DX**2+DY**2)
      COSA=DX/XL
      SINA=DY/XL
      EE(1)=E(1,2,MTYPE)
      EE(2)=E(1,3,MTYPE)
      COMM=EE(1)*EE(2)/XL
      S(1,1)=COSA*COSA*COMM
      S(1,2)=COSA*SINA*COMM
      S(1,3)=-S(1,1)
      S(1,4)=-S(1,2)
      S(2,1)=S(1,2)
      S(2,2)=SINA*SINA*COMM
      S(2,3)=-S(1,2)
      S(2,4)=-S(2,2)
      S(3,1)=S(1,3)
      S(3,2)=S(2,3)
      S(3,3)=S(1,1)
      S(3,4)=S(1,2)
      S(4,1)=S(1,4)
      S(4,2)=S(2,4)
      S(4,3)=S(3,4)
      S(4,4)=S(2,2)
      EP=E(1,4,MTYPE)*E(1,1,MTYPE)
      DX=DX*EP
      DY=DY*EP
      P(1)=S(1,1)*DX+S(1,2)*DY
      P(2)=S(2,1)*DX+S(2,2)*DY
  136 P(3)=-P(1)
      P(4)=-P(2)
      VOL=1.
      DO 140 I=1,4
      NPP=IX(N,I)
      RR(I)=R(NPP)
  140 ZZ(I)=Z(NPP)
      XC=0.5*(RR(1)+RR(2))
      YC=0.5*(ZZ(1)+ZZ(2))
      DO 200 I=1,3
      DO 200 J=1,8
  200 ST(I,J)=0.0
      DO 300 I=1,3
      DO 300 J=1,3
  300 C(I,J)=0.0
      DO 400 I=1,3
  400 EE(I)=E(1,I+1,MTYPE)
C
      RETURN
C
      END
      SUBROUTINE ELEMEN(R,Z,CODE,UU,CU,BI,BJ,T,SIGI,EPSI,SIGNM,BETA,
     .   RATIO,PR,B,A,IX,MTAG,NTAG,JNT,IBC,JBC)
      COMMON/ONE/ NUMNP,NUMEL,NUMMAT,NUMPC,NPC,MBAND,NUMBLK,NL,MTYPE,N,
     .   VOL,ACELR,ACELZ,Q,HED(18),STOP,SR,SR1,TOTAL,TOL,TOL1,XC,YC,
     .   TEMP,SIGN,SIGDI,SIGDJ,
     .   LLL,III,JJJ,JCK,KCHECK,JCHECK,NPLANE,NUMER,NCODE,JA,NEQ
     .   ,NBAND,NCHECK,NSTEP,N15,MT,NTOT
      COMMON/TWO/ C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),
     .   LM(4),EE(3),EPS(3)
      COMMON/THREE/  E(6,4,8),EO(4,8),IENS(8),XNU(8),RO(8),EPST(8),
     .   MIC(8),NTC(8),MID(8)
      DIMENSION R(NUMNP),Z(NUMNP),CODE(NUMNP),UU(NUMNP,2),CU(NEQ),
     .BI(NEQ),BJ(NEQ),T(NUMNP),SIGI(NUMEL,6),EPSI(NUMEL,6),SIGNM(NUMEL),
     .BETA(NUMEL),RATIO(NUMEL),PR(NPC,2),IX(NUMEL,5),MTAG(NUMEL),
     .NTAG(NUMEL),JNT(NUMEL),IBC(NPC),JBC(NPC)
      DIMENSION B(NL),A(NL,NBAND)
      DIMENSION U(3),V(3)
C
      EX=0.00001
      I=IX(N,1)
      J=IX(N,2)
      K=IX(N,3)
      L=IX(N,4)
      MTYPE=IX(N,5)
      VOL=0.
      TEMP=(T(I)+T(J)+T(K)+T(L))/4.0
      RAT=0.0
      NUMTC=NTC(MTYPE)
      IF (NUMTC.EQ.1) GO TO 100
      DO 50 M=2,NUMTC
      IF (E(M,1,MTYPE)-TEMP) 50,60,60
   50 CONTINUE
   60 DEN=E(M,1,MTYPE)-E(M-1,1,MTYPE)
      IF(DEN) 70,80,70
   70 RAT  =(TEMP-E(M-1,1,MTYPE))/DEN
   80 DO 90 KK=1,3
   90 EE(KK)=E(M-1,KK+1,MTYPE)+RAT  *(E(M,KK+1,MTYPE)-E(M-1,KK+1,MTYPE))
      GO TO 106
  100 DO 105 KK=1,3
  105 EE(KK)=E(1,KK+1,MTYPE)
  106 CONTINUE
C     ****************************************************************
C     * CALCULATE ORTHOTROPIC ELASTICITY MATRIX FOR MTAG=0,1,2       *
C     ****************************************************************
      KKK=MTAG(N)+1
      GO TO (117,107,108),KKK
  107 CONTINUE
C
C     MTAG=1
C
```

```
      EX=EEX
      EY=1.
      GO TO 109
  108 CONTINUE
C
C     MTAG=2
C
      EX=EEX
      EY=EEX
  109 CONTINUE
      GO TO (110,111),NPLANE
  110 CONTINUE
C
C     PLANE STRESS
C
      EE1=EE(1)*EX
      EE2=EE(1)*EY
      EEN=EE(2)*EY
      COMM=(1.-(EE1/EE2)*(EEN**2))
      C11=EE1/COMM
      C12=EEN*C11
      C21=C12
      C22=EE2/COMM
      GO TO 115
  111 CONTINUE
C
C     PLANE STRAIN
C
      EE1=EE(1)*EX
      EE2=EE(1)*EY
      EN1=EE(2)*EX
      EN2=EE(2)*EY
      EN3=EE(2)
      XA=1.-EN1*EN3
      XB=1.-EN2*EN3
      XC=1.+EN3
      XD=EN2*XC
      XE=EE2/EE1
      COMM=(XE*XA*XB-(EN2**2)*(XC**2))/EE2
      C11=XB/COMM
      C12=XD/COMM
      C21=C12
      C22=XE*XA/COMM
  115 CONTINUE
      EPSX=BETA(N)
      KCK=0
      IF(MID(MTYPE).EQ.0) GO TO 112
      EPSX=EO(2,MTYPE)
      IF(MTAG(N).NE.0) EPSX=EPSX-BETA(N)
  112 CONTINUE
```

```
ELEM101       S1= SIN(EPSX)
ELEM102       S2=S1*S1
ELEM103       C1= COS(EPSX)
ELEM104       C2=C1*C1
ELEM105       C1S1=C1*S1
ELEM106       C2S2=C2*S2
ELEM107       C4=C2**2
ELEM108       S4=S2*S2
ELEM109       DA=C11+C22
ELEM110       DB=C11-C12
ELEM111       DC=C22-C12
ELEM112       DD=2.*C2S2*C1.
ELEM113       C(1,1)=C4*C11+DD+S4*C22
ELEM114       C(1,2)=C2S2*DA+(C4+S4)*C12
ELEM115       C(1,3)=C1S1*(C2*DB-S2*DC)
ELEM116       C(2,2)=S4*C11+DD+C4*C22
ELEM117       C(2,3)=C1S1*(S2*DB-C2*DC)
ELEM118       C(3,3)=C2S2*(DB+DC)
ELEM119       IF((MID(MTYPE).EQ.0).OR.(KCK.EQ.1)) GO TO 116
ELEM120       IF(MTAG(N).NE.0) KCK=1
ELEM121       G=0.5*EE1/(1.+(EX**2)*EE2)
ELEM122       EPSX=2.*EPSX
ELEM123       S1=SIN(EPSX)
ELEM124       C1=COS(EPSX)
ELEM125       SC=S1*C1
ELEM126       S1=S1*S1
ELEM127       C1=C1*C1
ELEM128       C11=G*S1
ELEM129       C12=-C11
ELEM130       C13=-G*SC
ELEM131       C22=C11
ELEM132       C23=-C13
ELEM133       C33=G*C1
ELEM134       C(1,1)=C(1,1)+C11
ELEM135       C(1,2)=C(1,2)+C12
ELEM136       C(1,3)=C(1,3)+C13
ELEM137       C(2,2)=C(2,2)+C22
ELEM138       C(2,3)=C(2,3)+C23
ELEM139       C(3,3)=C(3,3)+C33
ELEM140       IF(KCK.EQ.0) GO TO 116
ELEM141       EX=EEX
ELEM142       EY=1.
ELEM143       IF(MTAG(N).EQ.1) GO TO 113
ELEM144       EY=EEX
ELEM145   113 CONTINUE
ELEM146       C11=EX*C(1,1)
ELEM147       C12=EX*C(1,2)
ELEM148       C22=EY*C(2,2)
ELEM149       C21=C12
ELEM150       EPSX=BETA(N)
```

```
ELEM151         GO TO 112
ELEM152     116 CONTINUE
ELEM153         C(2,1)=C(1,2)
ELEM154         C(3,2)=C(2,3)
ELEM155         C(3,1)=C(1,3)
ELEM156         IF(MTAG(N).EQ.0) GO TO 118
ELEM157         MTAG(N)=MTAG(N)+2
ELEM158         GO TO 118
ELEM159     117 CONTINUE
ELEM160         IF((MID(MTYPE).EQ.0).OR.(EO(1,MTYPE).EQ.1.)) GO TO 125
ELEM161         EX=EO(1,MTYPE)
ELEM162         EY=1.0
ELEM163         GO TO 109
ELEM164     125 CONTINUE
ELEM165 C       ******************************************************************
ELEM166 C       * CALCULATE ISOTROPIC ELASTICITY MATRIX                          *
ELEM167 C       ******************************************************************
ELEM168         EE1=EE(1)
ELEM169         EE2=EE(2)
ELEM170         IF(NPLANE.EQ.1) GO TO 119
ELEM171         EE1=EE1/(1.-EE2**2)
ELEM172         EE2=EE2/(1.-EE2)
ELEM173     119 CONTINUE
ELEM174         COMM=EE1/(1.-EE2**2)
ELEM175         C(1,1)=COMM
ELEM176         C(1,2)=COMM*EE2
ELEM177         C(1,3)=0.
ELEM178         C(2,1)=C(1,2)
ELEM179         C(2,2)=C(1,1)
ELEM180         C(2,3)=0.
ELEM181         C(3,1)=0.
ELEM182         C(3,2)=0.
ELEM183         C(3,3)=.5*COMM*(1.-EE2)
ELEM184     118 CONTINUE
ELEM185         DO 130 J=1,10
ELEM186         DO 120 I=1,3
ELEM187     120 ST(I,J)=0.
ELEM188         DO 130 I=1,10
ELEM189     130 S(I,J)=0.
ELEM190     135 CONTINUE
ELEM191         DO 140 I=1,4
ELEM192         NPP=IX(N,I)
ELEM193         RR(I)=R(NPP)
ELEM194     140 ZZ(I)=Z(NPP)
ELEM195         IF(IX(N,3)-IX(N,4)) 145,150,145
ELEM196     145 XC=(RR(1)+RR(2)+RR(3)+RR(4))/4.
ELEM197         YC=(ZZ(1)+ZZ(2)+ZZ(3)+ZZ(4))/4.
ELEM198         RR(5)=XC
ELEM199         ZZ(5)=YC
ELEM200         K=5
```

```
ELEM201          J=1
ELEM202          I=4
ELEM203          LM(3)=9
ELEM204          NT=4
ELEM205          GO TO 160
ELEM206      150 NT=1
ELEM207          LM(3)=5
ELEM208          I=1
ELEM209          J=2
ELEM210          K=3
ELEM211          XC=(RR(1)+RR(2)+RR(3))/3.
ELEM212          YC=(ZZ(1)+ZZ(2)+ZZ(3))/3.
ELEM213          RR(5)=RR(3)
ELEM214          ZZ(5)=ZZ(3)
ELEM215      160 DO 200 NN=1,NT
ELEM216          LM(1)=2*I-1
ELEM217          LM(2)=2*J-1
ELEM218          U(1)=ZZ(J)-ZZ(K)
ELEM219          U(2)=ZZ(K)-ZZ(I)
ELEM220          U(3)=ZZ(I)-ZZ(J)
ELEM221          V(1)=RR(K)-RR(J)
ELEM222          V(2)=RR(I)-RR(K)
ELEM223          V(3)=RR(J)-RR(I)
ELEM224          AREA=(RR(J)*U(2)+RR(I)*U(1)+RR(5)*U(3))/2.
ELEM225          VOL=VOL+AREA
ELEM226          COMM=.25/AREA
ELEM227          XNT=NT
ELEM228          COM=2./XNT
ELEM229          COM=COM*COMM
ELEM230          DO 180 L=1,3
ELEM231          II=LM(L)
ELEM232          ST(1,II)=ST(1,II)+U(L)*COM
ELEM233          ST(2,II+1)=ST(2,II+1)+V(L)*COM
ELEM234          ST(3,II)=ST(3,II)+V(L)*COM
ELEM235          ST(3,II+1)=ST(3,II+1)+U(L)*COM
ELEM236          DO 180 M=1,3
ELEM237          JJ=LM(M)
ELEM238          S(II,JJ)=S(II,JJ)+(C(1,1)*U(L)*U(M)+C(1,3)*(V(L)*U(M)+U(L)*V(M))
ELEM239         * +C(3,3)*V(L)*V(M))*COMM
ELEM240          S(II,JJ+1)=S(II,JJ+1)+(U(L)*(C(1,2)*V(M)+C(1,3)*U(M))+V(L)*(C(2,3)
ELEM241         *   *V(M)+C(3,3)*U(M)))*COMM
ELEM242          S(II+1,JJ+1)=S(II+1,JJ+1)+(C(2,2)*V(L)*V(M)+C(2,3)*(U(L)*V(M)+V(L)
ELEM243         * *U(M))+C(3,3)*U(L)*U(M))*COMM
ELEM244          S(JJ+1,II)=S(II,JJ+1)
ELEM245      180 CONTINUE
ELEM246          I=J
ELEM247          J=J+1
ELEM248      200 CONTINUE
ELEM249          IF(IX(N,3)-IX(N,4)) 220,250,220
ELEM250      220 DO 240 I=1,2
```

```
ELEM251       KK=10-I
ELEM252       DO 240 K=1,KK
ELEM253       IF(S(KK+1,KK+1).EQ.0.) S(KK+1,KK+1)=.000001
ELEM254       CC=S(KK+1,K)/S(KK+1,KK+1)
ELEM255       DO 230 J=1,3
ELEM256   230 ST(J,K)=ST(J,K)-CC*ST(J,KK+1)
ELEM257       DO 240 J=1,KK
ELEM258   240 S(J,K)=S(J,K)-CC*S(J,KK+1)
ELEM259   250 CONTINUE
ELEM260       RETURN
ELEM261 C
ELEM262       END
```

```
      SUBROUTINE STRESS(R,Z,CODE,UU,CU,BI,BJ,T,SIGI,EPSI,SIGNM,BETA,
     .   RATIO,PR,B,A,IX,MTAG,NTAG,JNT,IBC,JBC)
      COMMON/ONE/ NUMNP,NUMEL,NUMMAT,NUMPC,NPC,MBAND,NUMBLK,NL,MTYPE,N,
     .   VOL,ACELR,ACELZ,Q,HED(18),STOP,SR,SKI,TOTAL,TOL,TOLI,XC,YC,
     .   TEMP,SIGN,SIGDI,SIGDJ,
     .   LLE,III,JJJ,JCK,KCHECK,JCHECK,NPLANE,NUMER,NCODE,JA,NEQ
     .   ,NBAND,NCRACK,NSTEP,N15,M7,NTOT
      COMMON/TWO/ C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),
     .   LM(4),FE(3),EPS(3)
      COMMON/THREE/  E(8,4,8),EO(4,8),TENS(8),XNU(8),RO(8),EPST(8),
     .   MTC(8),NDC(8),MID(8)
      DIMENSION R(NUMNP),Z(NUMNP),CODE(NUMNP),UU(NUMNP,2),CU(NEQ),
     .BI(NEQ),BJ(NEQ),T(NUMNP),SIGI(NUMEL,6),EPSI(NUMEL,6),SIGNM(NUMEL),
     .BETA(NUMEL),RATIO(NUMEL),PR(NPC,2),IX(NUMEL,5),MTAG(NUMEL),
     .NTAG(NUMEL),JNT(NUMEL),IBC(NPC),JBC(NPC)
      DIMENSION B(NL),A(NL,NBAND)
      DO 600 M=1,NUMEL
      ID=M
      FIND(2'ID)
      IX(M,5)=IABS(IX(M,5))
      DO 50 I=1,6
   50 SIG(I)=0.0
C     ******************************************************************
C     * READ ELEMENT INFORMATION FROM FILE 2                           *
C     ******************************************************************
      READ(2'ID)  ((C(IIK,JJK),JJK=1,3),EE(IIK),IIK=1,3)
     1 ,((S(JJI,KKI),KKI=1,8),JJI=1,8),((ST(IKK,JKK),JKK=1,8),IKK=1,3)
     2 ,(RR(JII),ZZ(JII),JII=1,4),XC,YC,TEMP,VOL,MTYPE,N
      IF(IX(N,3)-IX(N,2)) 90,60,90
C
C     ******************************************************************
C     *    ONE-DIMENSIONAL ELEMENT                                     *
C     ******************************************************************
   60 IF(MTAG(N).EQ.4) GO TO 70
      I=IX(N,1)
      J=IX(N,2)
      DX=RR(2)-RR(1)
      DY=ZZ(2)-ZZ(1)
      XL= SQRT(DX**2+DY**2)
      DU=B(2*J-1)-B(2*I-1)
      DV=B(2*J)-B(2*I)
      DL=DV*DY/XL+DU*DX/XL
      SIG(1)=DL*EE(1)/XL
      IF(EE(2).EQ.0.) SIG(1)=0.
   70 SIGI(N,4)=SIG(1)
      SIGI(N,5)=0.
      SIGI(N,6)=0.
      GO TO 600
C
C     ******************************************************************
```

```
C     *     TWO-DIMENSIONAL ELEMENT                                    *
C     ****************************************************************
   90 MM=4
      IF(IX(N,3).EQ.IX(N,4)) MM=3
  170 DO 180 I=1,3
      RR(I)=0.
      DO 180 J=1,MM
      II=2*J
      JJ=2*IX(N,J)
  180 RR(I)=RR(I)+ST(I,II)*B(JJ)+ST(I,II-1)*B(JJ-1)
      DO 190 I=1,3
      DO 190 J=1,3
      SIG(I)=SIG(I)+C(I,J)*RR(J)
      IF(MTAG(N).EQ.4) SIG(I)=0.
  190 CONTINUE
      IF(MTAG(N).NE.3) GO TO 191
      EPSX=BETA(N)
      CC=COS(EPSX)
      SS=SIN(EPSX)
      S2=SS*SS
      C2=CC*CC
      SC=SS*CC
      SC2=2.*SC
      CSD=C2-S2
      SIGXX=C2*SIG(1)+S2*SIG(2)+SC2*SIG(3)
      SIGYY=S2*SIG(1)+C2*SIG(2)-SC2*SIG(3)
      SIGXY=-SC*(SIG(1)-SIG(2))+CSD*SIG(3)
      DX=FF(2)*SIGXX
      SIG(1)=SIG(1)-(C2*SIGXX+S2*DX-SC2*SIGXY)
      SIG(2)=SIG(2)-(S2*SIGXX+C2*DX+SC2*SIGXY)
      SIG(3)=SIG(3)-(SC*(SIGXX-DX    )+CSD*SIGXY)
  191 CONTINUE
      DO 192 I=1,3
      SIG1(N,I+3)=SIG(I)
      EPS1(N,I+3)=RR(I)
  192 CONTINUE
  600 CONTINUE
      RETURN
      END
      SUBROUTINE GRIFTH (MTAG,JNT,RATIO,BETA,SIGI,SIGNM)
C
      COMMON/ONE/ NUMNP,NUMEL,NUMMAT,NUMPC,NPC,MBAND,NUMBLK,NL,MTYPE,N,
     .   VOL,ACELR,ACELZ,Q,HED(18),STOP,SR,SR1,TOTAL,TOL,TOLI,XC,YC,
     .   TEMP,SIGN,SIGDI,SIGDJ,
     .   LLL,III,JJJ,JCK,KCHECK,JCHECK,NPLANE,NUMER,NCODE,JA,NEQ
     .   ,NBAND,NCRACK,NSTEP,N15,M7,NTOT
      COMMON/TWO/ C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),
     .   LM(4),EE(3),EPS(3)
      COMMON/THREE/  E(8,4,8),EO(4,8),TENS(8),XNU(8),RO(8),EPST(8),
     .   MTC(8),NTC(8),MID(8)
      DIMENSION MTAG(NUMEL),JNT(NUMEL),RATIO(NUMEL),BETA(NUMEL),
     .   SIGI(NUMEL,6),SIGNM(NUMEL)
C    *********************************************************
C     * XNU,XN= COEF. OF FRICTION FOR THE MODIFIED GRIFFITH CRITERION  *
C     * TENS,SIGT= TENSILE STRENGTH OF THE MATERIAL                    *
C     * SIGN= STRESS COMPONENT NORMAL TO THE FRACTURE PLANE            *
C     * EPSX= ANGLE BETWEEN P AND SIGN                                 *
C     * BETA= ANGLE BETWEEN X-AXIS AND SIGN                            *
C     * ANGLES ARE POSITIVE COUNTERCLOCKWISE                           *
C     ******************************************************************
      MTAGI=MTAG(N)
      JNTI=JNT(N)
      SIGYY=0.
      SIGN=0.
      IF(MTAG(N).LT.2) GO TO 600
      MTAG(N)=0
      JNT(N)=0
      BETA(N)=0.
      PP=SIG(4)
      Q=SIG(5)
      R=SIG(6)
      SIGT=TENS(MTYPE)
      XN=XNU(MTYPE)
      SUM=3.0*PP+Q
      TOL=0.005*SIGT
      TOLL=TOL
      IF(JCK.EQ.0) TOLL=0.
      MD=MID(MTYPE)
      IF((MTAGI.EQ.2).AND.(JNTI.LT.4)) GO TO 30
C     CHECK INITIAL WEAK PLANE
      IF(MD.EQ.0) GO TO 30
      WX=2.*(EO(2,MTYPE)-R/57.296)
      W=0.5*WX
      X=COS(WX)
      PQADD=PP+Q
      PQSUB=PP-Q
      SIGYY=0.5*(PQADD+PQSUB*X)
      SIGT=EO(4,MTYPE)
      XN=EO(3,MTYPE)
      IF(SIGYY.LE.0.) GO TO 200
      SIGN=SIGYY
      GO TO 40
   30 CONTINUE
      IF(SUM.LT.0.) GO TO 100
      SIGN=PP
   40 CONTINUE
      DSIG=SIGN-SIGI
      IF((JCK.EQ.1).AND.(ABS(DSIG).LT.TOLL)) GO TO 50
      IF(DSIG.LT.TOLL) GO TO 600
C     *****************************************************************
C     * FRACTURE PLANE COINCIDES WITH SIG(5) IF CRITICAL ANGLE IS ZERO *
C     * APPLY ORIGINAL GRIFFITH CRITERION                              *
C     *****************************************************************
   50 CONTINUE
      IF(MD.EQ.0) GO TO 70
      IF((MTAGI.EQ.2).AND.(JNTI.LT.4)) GO TO 70
      MTAG(N)=1
      JNT(N)=4
      BETA(N)=ED(2,MTYPE)
      GO TO 320
   70 JNT(N)=1
      SIGN=PP
      SIGYY=SIGN
      EPSX=0.0
      GO TO 300
  100 CONTINUE
C     *****************************************************************
C     * CRITICAL ANGLE IS NOT ZERO                                    *
C     * APPLY THE ORIGINAL GRIFFITH IF THE INITIAL FLAW IS OPEN       *
C     *****************************************************************
      PQADD=PP+Q
      PQSUB=PP-Q
      X=-0.5*PQSUB/PQADD
      SIGYY=0.5*(PQADD+PQSUB*X)
      IF(SIGYY.LT.0.) GO TO 200
      SIGN=-PQSUB**2/(8.*PQADD)
      DSIG=SIGN-SIGI
      IF((JCK.EQ.1).AND.(ABS(DSIG).LT.TOLL)) GO TO 150
      IF(DSIG.LT.TOLL) GO TO 600
  150 CONTINUE
      WX= ARCOS(X)
      W=0.5*WX
      JNT(N)=2
      Z=Q/PP
      S1= SIN(W)
      S2=S1**2
      SS= SIN(WX)
      ZZZ=Z*Z-1.0
      Z=Z-1.0
      ARG=(Z*S2+1.0- SQRT(ZZZ*S2+1.0))/(0.5*Z*SS)
      EPSX= ATAN(ARG)-W
      GO TO 300
  200 CONTINUE
C     ****************************************************************
C     * APPLY THE MODIFIED GRIFFITH IF THE INITIAL FLAW IS CLOSED     *
C     ****************************************************************
      IF(MO.EQ.0) GO TO 240
C     CHECK SHEAR STRENGTH
      SIGN=0.5*(PQSUB*SQRT(1.+XN**2)+PQADD*XN)
      DSIG=SIGN-SIGT
      IF((JCK.EQ.1).AND.(ABS(DSIG).LT.TOLL)) GO TO 220
      IF(DSIG.LT.TOLL) GO TO 600
  220 SIGM=.5*PQADD
      TAU=.5*PQSUB
      IF(TAU.EQ.0.) GO TO 600
      WX=ATAN(XN)
      X=((SIGM+SIGT/XN)/TAU)*SIN(WX)
      IF(X.GT.1.) X=1.
      AX=ARSIN(X)
      B1=-.5*(AX+WX)
      B2=.5*(3.1416+AX-WX)
      BW=.5*ATAN2(1.,XN)
C     CHECK BOUNDS OF ANGLE
      IF((BW.GE.B1).AND.(BW.LE.B2)) GO TO 230
      GO TO 600
  230 XN=XNU(MTYPE)
      SIGM=.25*(PQSUB*SQRT(1.+XN**2)+PQADD*XN)
      IF(SIGN.GE.SIGM) GO TO 240
      JNT(N)=5
      GO TO 270
  240 CONTINUE
      X=1.0/XN
      WX= ATAN(X)
      SIGYY=0.5*(PQADD+PQSUB* COS(WX))
      IF(SIGYY.GE.0.1) GO TO 600
      W=0.5*WX
      SIGN=0.25*(PQSUB* SQRT(1.+XN**2)+PQADD*XN)
      DSIG=SIGN-SIGT
      IF((JCK.EQ.1).AND.(ABS(DSIG).LT.TOLL)) GO TO 250
      IF(DSIG.LT.TOLL) GO TO 600
  250 CONTINUE
      JNT(N)=3
  270 CONTINUE
      EPSX=0.7854-W
  300 CONTINUE
      MTAG(N)=1
      BETA(N)=R/57.296-EPSX
      ANG=57.296*BETA(N)
      IF(ANG.GT.90.0) BETA(N)=BETA(N)-3.1416
      IF(ANG.LT.-90.) BETA(N)=BETA(N)+3.1416
  320 CONTINUE
      IF(KCHECK.EQ.0) GO TO 600
C     ******************************************************************
C     *   EVALUATE STRESS RATIO FOR EACH ELEMENT                       *
C     ******************************************************************
      AO=SIGI(N,1)+SIGI(N,2)
      BO=SIGI(N,1)-SIGI(N,2)
      A1=SIG(1)+SIG(2)
      B1=SIG(1)-SIG(2)
      CO=BO**2+4.0*SIGI(N,3)**2
      C1=BO*B1+4.0*SIGI(N,3)*SIG(3)
      C2=B1**2+4.0*SIG(3)**2
      IJNT=JNT(N)+1
      GO TO (600,350,400,500,340,500),IJNT
  340 C4=X**4
      C1=C1*C4
      C2=C2*C4
  350 SIGT2=2.*SIGI
      DSIGT=SIGT2-AO
      AAA=A1**2-C2
      BB=-A1*DSIGT-C1
      CC=DSIGT**2-CO
      GO TO 550
  400 CONTINUE
      AAA=C2
      BB=C1+4.0*SIGT*A1
      CC=8.0*SIGT*AO+CO
      GO TO 550
  500 CONTINUE
      XN2=XN**2
      RXN2=1.0+XN2
      AAA=RXN2*C2-XN2*A1**2
      IF(JNT(N).EQ.3) GO TO 520
      BB=RXN2*C1-XN2*A1*AO+2.0*SIGT*A1*XN
      CC=RXN2*CO-XN2*AO*AO+4.0*SIGT*AO*XN- 4.0*SIGT**2
      GO TO 550
  520 BB=RXN2*C1-XN2*A1*AO+4.0*SIGT*A1*XN
      CC=RXN2*CO-XN2*AO*AO+8.0*SIGT*AO*XN-16.0*SIGT**2
  550 CONTINUE
      DD=BB**2-AAA*CC
      IF(DD.GE.0.0) GO TO 560
      GO TO 600
  560 ROOT= SQRT(DD)
      RI=(-BB+ROOT)/AAA
      RJ=(-BB-ROOT)/AAA
      RATIO(N)=RI
      IF((RATIO(N).LT.TOLI ).AND.(RJ.GE.TOLI )) RATIO(N)=RJ
      IF((RATIO(N).GT.1.).AND.(RJ.LT.1.)) RATIO(N)=RJ
      IF( ABS(1.-RATIO(N)).LE.TOLI ) RATIO(N)=1.0
      IF(RATIO(N).LT.TOLI) RATIO(N)=TOLI
      IF (RATIO(N).GT.1.) RATIO(N)=1.
  600 CONTINUE
      SIGNM(N)=SIGN
      IF(MTAG(N).NE.MTAGI) JCHECK=1
      IF(MTAGI.EQ.2) MTAG(N)=2
      IF(MTAG(N).NE.1) RATIO(N)=1.0
      RETURN
C
      END
      SUBROUTINE SCALE(IX,MTAG,NTAG, JNT,RATIO,BETA,SIGI,SIGNM,EPSI,CU,
     . B,A)
C
      COMMON/ONE/ NUMNP,NUMEL,NUMMAT,NUMPC,NPC,MBAND,NUMBLK,NL,MTYPE,N,
     .   VOL,ACELR,ACELZ,Q,HED(18),STOP,SR,SR1,TOTAL,TOL,TOLI,XC,YC,
     .   TEMP,SIGN,SIGDI,SIGDJ,
     .   LLL,III,JJJ,JCK,KCHECK,JCHECK,NPLANE,NUMER,NCODE,JA,NEQ
     .   ,NBAND,NCRACK,NSTEP,N15,M7,NTOT
      COMMON/TWO/ C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),
     .   LM(4),EE(3),EPS(3)
      COMMON/THREE/  E(8,4,8),EO(4,8),TENS(8),XNU(8),RO(8),EPST(8),
     .   MTC(8),NTC(8),MID(8)
      DIMENSION IX(NUMEL,5),MTAG(NUMEL),NTAG(NUMEL),JNT(NUMEL),RATIO(NUM
     .EL),BETA(NUMEL),SIGI(NUMEL,6),SIGNM(NUMEL),EPSI(NUMEL,6),CU(NEQ)
      DIMENSION B(NL),A(NL,N8AND)
      DIMENSION UX(8),UY(8),F(4)
      KIK=0
      JCK=1
      ICHK=1
      ICHECK=0
      IF(III.GT.0) GO TO 350
      JCK=0
      KCHECK=1
      KKK=0
      SRR=1.
      SR1=1.0
      WRITE(6,2005) LLL
      GO TO 600
  100 CONTINUE
      NKT=0
      KKK=KKK+1
      SR=1.0
      DO 200 M=1,NUMEL
  200 IF(RATIO(M).LT.SR) SR=RATIO(M)
      SR3=SR
      TOLL=0.
      DO 210 M=1,NUMEL
      IF(MTAG(M).NE.1) GO TO 210
      MTYPE=IX(M,5)
      TOLJ=0.005
      TOLK=ABS(RATIO(M)-SR)
      IF(TOLK.GT.TOLJ) GO TO 210
      IF(TOLK.GT.TOLL) TOLL=TOLK
  210 CONTINUE
      IF(SR3.LT.1.) SR3=SR+TOLL
      IF(SR3.GT.1.) SR3=1.0
      DO 220 M=1,NUMEL
  220 IF( ABS(RATIO(M)-SR).LE.TOLJ ) RATIO(M)=SR3
      SR=SR3
      IF(SR3.NE.1.0) GO TO 240
      KCHECK=0
      SR=SR*SR1
      GO TO 250
C
C     INTERPOLATE OR EXTRAPOLATE SCALING FACTOR
C
  240 CALL        INTER (IX,MTAG,JNT,RATIO,BETA,SIGI,SIGNM,EPSI,CU,B,A)
  250 SR1=SR
      DO 280 M=1,NUMEL
      IF(MTAG(M).GT.1) GO TO 280
      IF(MTAG(M).EQ.1) NKT=NKT+1
      IF(RATIO(M).EQ.SR3) GO TO 280
      IF(MTAG(M).EQ.1) NKT=NKT-1
      RATIO(M)=1.0
      BETA(M)=0.
      JNT(M)=0
      MTAG(M)=0
  280 CONTINUE
      IF(KCHECK.NE.0) GO TO 345
      KKK=KKK-1
      SR=SR*(1.-TOTAL)
      KIK=0
      IF((KKK.EQ.0).AND.(SR.EQ.1.)) KIK=1
      WRITE(6,2006)
      WRITE(6,2002) LLL,SR,KKK
      GO TO 350
  345 WRITE(6,2003) LLL,KKK,SR
      IF(SR.GE.0.0001) GO TO 350
      WRITE(6,2004) SR
      SR=0.
      RETURN
  350 SRR=SR1
      IF(III.GT.0) SRR=1.0
      GO TO 600
  352 IF((ICHK.EQ.0).AND.(KCHECK.EQ.1)) GO TO 100
C     *****************************************************************
C     * SCALE DOWN DISPLACEMENTS                                      *
C     *****************************************************************
      IF((III.EQ.0).AND.(KCHECK.EQ.0)) GO TO 357
      GO TO 358
  357 TOTAL=TOTAL+SR
      IF((TOTAL.GE..999).OR.(SR.EQ.0.0)) TOTAL=1.0
  358 CONTINUE
      IF((III.GT.0).AND.(ICHECK.EQ.0)) GO TO 550
      IF(KIK.EQ.1) GO TO 359
      WRITE(6,2006)
      WRITE(6,2000)
  359 DO 500 M=1,NUMNP
      N1=2*M-1
      N2=N1+1
      BX=SRR*B(N1)
      BY=SRR*B(N2)
      IF(ICHECK.EQ.1) GO TO 360
      BXT=BX+CU(N1)
      BYT=BY+CU(N2)
      GO TO 370
  360 CU(N1)=CU(N1)+BX
      CU(N2)=CU(N2)+BY
      BXT=CU(N1)
      BYT=CU(N2)
  370 IF(KIK.EQ.1) GO TO 500
      WRITE(6,1000) M,BX,BY,BXT,BYT
  500 CONTINUE
      KIK=0
  550 CONTINUE
      ICHK=0
      IF(III.NE.0) GO TO 900
      IF(KCHECK.EQ.0) GO TO 900
      GO TO 100
C     ******************************************************************
C     * SCALE DOWN STRESSES                                            *
C     ******************************************************************
  600 CONTINUE
      SM=0.
      JCHECK=0
      KCK=0
      IF((KKK.GT.0).AND.(KCHECK.EQ.1)) KCK=1
      IF((KIK.EQ.1).OR.(KCK.EQ.1)) GO TO 605
      WRITE(6,2001)
  605 CONTINUE
      DO 800 M=1,NUMEL
      ID=M
C     ******************************************************************
C     * READ ELEMENT INFORMATION FROM FILE 2                           *
C     ******************************************************************
      READ(2'ID)  ((C(IIK,JJK),JJK=1,3),EE(IIK),IIK=1,3)
     1 ,((S(JJI,KKI),KKI=1,8),JJI=1,8),((ST(IKK,JKK),JKK=1,8),IKK=1,3)
     2 ,(RR(JII),ZZ(JII),JII=1,4),XC,YC,TEMP,VOL,MTYPE,N
      DO 640 I=1,3
  640 SIG(I)=SRR*SIGI(M,I+3)+SIGI(M,I)
      IF(IX(M,3).NE.IX(M,2)) GO TO 650
      DX=RR(2)-RR(1)
      DY=ZZ(2)-ZZ(1)
      SIG(6)=90.
      IF(DX.NE.0.) SIG(6)=57.296*ATAN2(DY,DX)
      SIG(4)=SIG(1)
      SIG(5)=0.
      GO TO 765
  650 CC=(SIG(1)+SIG(2))/2.0
      BB=(SIG(1)-SIG(2))/2.0
```

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SCAL151       CR= SQRT(BB**2+SIG(3)**2)
SCAL152       SIG(4)=CC+CR
SCAL153       SIG(5)=CC-CR
SCAL154       SIG(6)=0.0
SCAL155       IF((BB.EQ.0.).AND.(SIG(3).EQ.0.)) GO TO 660
SCAL156       SIG(6)=28.648* ATAN2(SIG(3),BB)
SCAL157   660 MTYPE=IX(M,5)
SCAL158       IF(MTC(MTYPE).EQ.0) GO TO 765
SCAL159       IF(III.EQ.0) GO TO 755
SCAL160       IF((MTAG(M).EQ.3).AND.(SIG(4).GT.    TENS(MTYPE))) MTAG(M)=2
SCAL161       IF(MTAG(M).EQ.2) JCHECK=1
SCAL162       IF(MTAG(M).GT.2) GO TO 765
SCAL163       CALL       GRIFTH (MTAG,JNT,RATIO,BETA,SIGI,SIGNM)
SCAL164       GO TO 765
SCAL165   755 CONTINUE
SCAL166       IF(KCHECK.EQ.1) GO TO 757
SCAL167       IF((MTAG(M).EQ.3).AND.(SIG(4).GT.    TENS(MTYPE))) MTAG(M)=2
SCAL168       IF((MTAG(M).EQ.1).OR.(MTAG(M).EQ.2)) JCHECK=1
SCAL169       IF(MTAG(M).GT.2) GO TO 765
SCAL170   757 CALL       GRIFTH (MTAG,JNT,RATIO,BETA,SIG1,SIGNM)
SCAL171   765 SN=0.
SCAL172       IF(KCHECK.EQ.1) GO TO 770
SCAL173       IF((MTAG(M).EQ.0).OR.(MTAG(M).GT.2)) GO TO 770
SCAL174       SN=SIGNM(M)/TENS(MTYPE)
SCAL175       IF(SIGNM(M).GT.SM) SM=SIGNM(M)
SCAL176   770 IF(KCK.EQ.1) GO TO 800
SCAL177       IF((SN.GE.1.).OR.(RATIO(M).GT.1.)) RATIO(M)=1.
SCAL178       IF(KIK.EQ.1) GO TO 800
SCAL179       ANG=57.296*BETA(M)
SCAL180       WRITE(6,1001) M,XC,YC,(SIG(I),I=1,6),MTAG(M),JNT(M),ANG,RATIO(M)
SCAL181      *,SN
SCAL182   800 CONTINUE
SCAL183       IF(III.NE.0) GO TO 830
SCAL184       IF(KCHECK.NE.0) GO TO 828
SCAL185       IF(JJJ.NE.0) GO TO 828
SCAL186 C
SCAL187 C     ******************************************************************
SCAL188 C     * CHECK CRACK CLOSURE WHEN III=0,KCHECK=0 & JJJ=0  .             *
SCAL189 C     ******************************************************************
SCAL190 C
SCAL191       WRITE(6,2007)
SCAL192       DO 827 N=1,NUMEL
SCAL193       IF(MTAG(N).LT.3) GO TO 827
SCAL194       IF(IX(N,2).EQ.IX(N,3)) GO TO 827
SCAL195       ID=N
SCAL196       MM=4
SCAL197       IF(IX(N,1).EQ.IX(N,4)) MM=3
SCAL198       FIND(2'ID)
SCAL199       KTAG=0
SCAL200       DO 802 I=1,MM
```

```
      UX(I)=0.
  802 UY(I)=0.
      READ(2'ID)  ((C(IIK,JJK),JJK=1,3),EE(IIK),IIK=1,3)
     1 ,((S(JJI,KKI),KKI=1,8),JJI=1,8),((SI(IKK,JKK),JKK=1,8),IKK=1,3)
     2 ,(RR(JII),ZZ(JII),JII=1,4),XC,YC,TEMP,VOL,MTYPE,N
      ICK=0
      EPT=EPST(MTYPE)
C
C     DECOMPOSE JOINT OPENING INTO X AND Y COMPONENTS
C
      EPSX=1.5708+BETA(N)
      IF(EPSX.GE.1.5708) EPSX=1.56
  804 SLOPE=TAN(EPSX)
      DO 806 I=1,MM
  806 F(I)=SLOPE*(RR(I)-XC)-(ZZ(I)-YC)
      EPSX=BETA(N)
      IF(BETA(N).LT.0.) EPSX=3.1416+EPSX
      IF(KTAG.EQ.1) EPSX=EPSX-1.5708
      DUX=EPT*COS(EPSX)
      DUY=EPT*SIN(EPSX)
      DO 808 I=1,MM
      IF(F(I).LT.0.) GO TO 808
      UX(I)=UX(I)+DUX
      UY(I)=UY(I)+DUY
  808 CONTINUE
      IF(KTAG.EQ.1) GO TO 810
      IF(MTAG(N).EQ.3) GO TO 812
      KTAG=1
      EPSX=BETA(N)
      GO TO 804
  810 EPSX=EPSX+1.5708
  812 CONTINUE
C
C     EVALUATE EQUIVALENT STRAINS
C
      DO 816 I=1,3
      RR(I)=0.0
      DO 816 J=1,MM
      K=2*J
  816 RR(I)=RR(I)+ST(I,K)*UY(I)+ST(I,K-1)*UX(I)
  817 CONTINUE
      C1=COS(EPSX)
      S1=SIN(EPSX)
      SC=C1*S1
      C1=C1*C1
      S1=S1*S1
      IF(ICK.EQ.1) GO TO 818
      DP=SC*RR(3)
      EPT1=ABS(C1*RR(1)+S1*RR(2)+DP)
      EPT2=ABS(S1*RR(1)+C1*RR(2)-DP)
      IF(BETA(N).GT.0.) GO TO 818
      EPSX=BETA(N)
      ICK=1
      GO TO 817
C
C     EVALUATE ELEMENT PRINCIPAL STRAINS, CHECK CLOSURE POSSIBILITY
C
  818 EPS(1)=SRR*EPSI(N,4)+EPSI(N,1)
      EPS(2)=SRR*EPSI(N,5)+EPSI(N,2)
      EPS(3)=SRR*EPSI(N,6)+EPSI(N,3)
      DP=SC*EPS(3)
      EPS1=C1*EPS(1)+S1*EPS(2)+DP
      EPS2=S1*EPS(1)+C1*EPS(2)-DP
      C1=EPT1+EPS1
      S1=EPT2+EPS2
      IF(C1.GT.0.) GO TO 824
      IF(S1.GT.0.) GO TO 823
      MTAG(N)=0
      GO TO 826
  823 MTAG(N)=MTAG(N)-3
      IF(MTAG(N).EQ.0) GO TO 826
      BETA(N)=BETA(N)+1.5708
      IF(BETA(N).GT.1.5708) BETA(N)=BETA(N)-3.1416
      GO TO 826
  824 IF(S1.GT.0.) GO TO 827
      IF(MTAG(N).EQ.3) GO TO 827
      MTAG(N)=1
  826 JCHECK=1
      NTAG(N)=1
      ANG=57.296*BETA(N)
      WRITE(6,2015)
      WRITE(6,2020) N,MTAG(N),ANG,EPT,EPT1,EPT2,EPS1,EPS2,C1,S1
  827 CONTINUE
  828 IF(IDTAL.EQ.1.) ICHECK=1
      IF((KCHECK.EQ.0).AND.(NKT.LE.0)) ICHECK=1
      GO TO 840
C
C     SEARCH FOR MAX. SIGNM
C
  830 DO 835 N=1,NUMEL
      IF( MTAG(N).GT.2) GO TO 835
      DN=ABS(SIGNM(N)-SM)
      IF(DN.LE.TOL) GO TO 835
      IF(MTAG(N).NE.2) GO TO 834
      MTAG(N)=3
      DA=1.5708
      IF(BETA(N).GT.0.) DA=-DA
      BETA(N)=BETA(N)+DA
      GO TO 835
  834 MTAG(N)=0
```

```
      JNT(N)=0
      BETA(N)=0.
      RATIO(N)=1.
  835 ICHECK=1
  840 IF(JCHECK.EQ.1) ICHECK=0
      IF(ICHECK.EQ.1) GO TO 850
      GO TO 352
  850 DO 860 M=1,NUMEL
      DO 860 I=1,3
      SIGI(M,I)=SRR*SIGI(M,I+3)+SIGI(M,I)
      EPSI(M,I)=SRR*EPSI(M,I+3)+EPSI(M,I)
  860 CONTINUE
      GO TO 352
  900 CONTINUE
      WRITE(6,2010) TOTAL
      RETURN
 1000 FORMAT(I12,4E20.7)
 1001 FORMAT(I7,2F8.2,1P5E12.4,0P1F7.2,2I5,F7.2,1X,F7.4,F7.2)
 2000 FORMAT(12H0N.P.NUMBER ,17X,3HDUX,17X,3HDUY,18X,2HUX,18X,2HUY)
 2001 FORMAT (7H0EL.NO.  7X 1HX  7X 1HY 4X 8HX-STRESS 4X 8HY-STRESS 3X
     1 9HXY-STRESS,2X,10HMAX-STRESS,2X,10HMIN-STRESS,7H  ANGLE
     2,1X,4HMTAG,2X,3HJNT,3X,4HBETA,2X,5HRATIO,2X,5HSIGNM)
 2002 FORMAT(/ ' LOAD INCREMENT NO.',I5,/ ' LOAD APPLIED AS A FRACTION O
     *F TOTAL LOAD =',F9.5,/' NUMBER OF BETA STABILITY ITERATIONS=',I5/)
 2003 FORMAT(1H0,' LOAD INCREMENT NO.',I5,'   STABILITY ITERATION NO.',
     * I3,'   SCALING FACTOR =',G10.4)
 2004 FORMAT(//' SR=',G10.4,'  CALL EXIT')
 2005 FORMAT(1H1,' INITIAL STATE FOR LOAD INCREMENT',I5)
 2006 FORMAT(1H1)
 2007 FORMAT(//  ' CHECK CRACK OPENING STRAINS ')
 2010 FORMAT(1H0,' LOAD ACCUMULATED AS A FRACTION OF THE TOTAL IS',G15.5
     *//)
 2015 FORMAT(1H0,' N  TAG',8X,'BETA',8X,' EPT',8X,'EPT1',8X,'EPT2',8X,
     *'EPS1',8X,'EPS2',7X,'DEPS1',7X,'DEPS2')
 2020 FORMAT(I3,I5,F12.2,6E12.4)
 2025 FORMAT(I8,2F10.4,3E12.5)
      END
      SUBROUTINE  INTER (IX,MTAG,JNT,RATIO,BETA,SIGI,SIGNM)
C
      COMMON/ONE/ NUMNP,NUMEL,NUMMAT,NUMPC,NPC,MBAND,NUMBLK,NL,MTYPE,N,
     .   VOL,ACELR,ACELZ,Q,HED(18),STOP,SR,SR1,TOTAL,TOL,TOLI,XC,YC,
     .   TEMP,SIGN,SIGDI,SIGDJ,
     .   LLL,III,JJJ,JCK,KCHECK,JCHECK,NPLANE,NUMFR,NCODE,JA,NEQ
     .   ,NBAND,NCRACK,NSTEP,N15,M7,NTOT
      COMMON/TWO/ C(3,3),S(10,10),SIG(6),P(8),ST(3,10),RR(5),ZZ(5),
     .   LM(4),LE(3),EPS(3)
      COMMON/THREE/  E(6,4,8),EO(4,8),TENS(8),XNU(8),RO(8),EPST(8),
     .   MTC(8),NIC(8),MID(8)
      DIMENSION IX(NUMEL,5),MTAG(NUMEL),JNT(NUMEL),RATIO(NUMEL),
     .   BETA(NUMEL),SIGI(NUMEL,6),SIGNM(NUMEL)
      JCK=1
      SRR=SR1
      IJJ=0
      DO 200 N=1,NUMEL
      IF(RATIO(N).NE.SR) GO TO 200
      SR3=RATIO(N)
      BETAI=BETA(N)
      MTAGI=MTAG(N)
      JNTI=JNT(N)
      SR=SR*SR1
      MTYPE=IX(N,5)
      SIGT=TENS(MTYPE)
      IF(IX(N,2).EQ.IX(N,3)) GO TO 200
   50 CONTINUE
      DO 100 I=1,3
  100 SIG(I)=SRR*SIGI(N,I+3)+SIGI(N,I)
      CC=(SIG(1)+SIG(2))/2.0
      BB=(SIG(1)-SIG(2))/2.0
      CR= SQRT(BB**2+SIG(3)**2)
      SIG(4)=CC+CR
      SIG(5)=CC-CR
      SIG(6)=0.0
      IF((BB.EQ.0.).AND.(SIG(3).EQ.0.)) GO TO 150
      SIG(6)=28.648* ATAN2(SIG(3),BB)
  150 CONTINUE
      CALL       GRIFTH (MTAG,JNT,RATIO,BETA,SIGI,SIGNM)
      IF(IJJ.GT.0) GO TO 300
      SIGDI=SIGN-SIGT
      SRR=SR
      IF(SRR.LT.TOLI) SRR=TOLI
      IJJ=1
      GO TO 50
  200 CONTINUE
  300 CONTINUE
      IF (IX(N,2).EQ.IX(N,3)) GO TO 400
      RATIO(N)=SR3
      BETA(N)=BETAI
      MTAG(N)=MTAGI
      JNI(N)=JNTI
      SIGDJ=SIGN-SIGT
      IF(( ABS(SIGDI).LE.TOL   ).OR.( ABS(SIGDJ).LE.TOL   )) GO TO 350
      SR=(SR1*SIGDJ-SR*SIGDI)/(SIGDJ-SIGDI)
      GO TO 400
  350 CONTINUE
      IF( ABS(SIGDI).GT.TOL   ) GO TO 370
      SR=SR1
      GO TO 390
  370 IF( ABS(SIGDJ).GT.TOL   ) GO TO 400
      SR=SRR
  390 CONTINUE
  400 CONTINUE
      IF(SR.GE.0.99 ) SR=1.0
      IF(SR.LT.0.001) SR=0.001
      IF(SR.EQ.1.0) KCHECK=0
      TOLJ=0.01
      DA=SR-SRR
      DB=SR-SR1
      IF((ABS(DA).LE.TOLJ).OR.(ABS(DB).LE.TOLJ)) KCHECK=0
      WRITE(6,1000) SR1,SRR,SR
      RETURN
 1000 FORMAT(//' OLD SR=',G14.4,' NEW SR=',G14.4, 10X,' INTERPOLATED SR=
     *',G14.4//)
C
      END
      SUBROUTINE MODIFY(A,B,NEQ,MBAND,NBAND,N,U)
C
C
      DIMENSION B(NEQ),A(NEQ,NBAND)
      DO 250 M=2,MBAND
      K=N-M+1
      IF(K) 235,235,230
  230 B(K)=B(K)-A(K,M)*U
      A(K,M)=0.0
  235 K=N+M-1
      IF(NEQ-K) 250,240,240
  240 B(K)=B(K)-A(N,M)*U
      A(N,M)=0.0
  250 CONTINUE
      A(N,1)=1.0
      B(N)=U
      RETURN
      END
      SUBROUTINE BANSOL (B,A,MM,NBAND,NUMBLK,NLL,JA)
C
C
      DIMENSION B(NLL),A(NLL,NBAND)
      NN=NLL/2
      NL=NN+1
      NH=NN+NN
      NB=0
      NBK=1
      FIND(1'1)
      GO TO 150
  100 NB=NB+1
      DO 125 N=1,NN
      NM=NN+N
      B(N)=B(NM)
      B(NM)=0.0
      DO 125 M=1,MM
      A(N,M)=A(NM,M)
  125 A(NM,M)=0.0
      IF (NUMBLK-NB) 150,200,150
  150 READ (1'NBK) (B(N),(A(N,M),M=1,MM),N=NL,NH)
      NNK=NBK
      NBK=NBK+JA
      IF (NB) 200,100,200
  200 DO 300 N=1,NN
      IF (A(N,1)) 225,300,225
  225 B(N)=B(N)/A(N,1)
      DO 275 L=2,MM
      IF (A(N,L)) 230,275,230
  230 C=A(N,L)/A(N,1)
      I=N+L-1
      J=0
      DO 250 K=L,MM
      J=J+1
  250 A(I,J)=A(I,J)-C*A(N,K)
      B(I)=B(I)-A(N,L)*B(N)
      A(N,L)=C
  275 CONTINUE
  300 CONTINUE
      NBK=NNK-JA
      IF(NUMBLK.EQ.NB) GO TO 410
      WRITE (1'NBK) (B(N),(A(N,M),M=2,MM),N=1,NN)
      NBK=NNK+JA
      GO TO 100
  410 DO 450 M=1,NN
      N=NN+1-M
      DO 425 K=2,MM
      L=N+K-1
  425 B(N)=B(N)-A(N,K)*B(L)
      NM=N+NN
      B(NM)=B(N)
  450 A(NM,NB)=B(N)
      NB=NB-1
      IF(NB.EQ.0) GO TO 500
      READ  (1'NBK)  (B(N),(A(N,M),M=2,MM),N=1,NN)
      NBK=NBK-JA
      GO TO 410
  500 K=0
      DO 600 NB=1,NUMBLK
      DO 600 N=1,NN
      NM=N+NN
      K=K+1
  600 B(K)=A(NM,NB)
      RETURN
C
      END
```